Some Examples of Using Bayesian Statistics in Modeling Human Cognition

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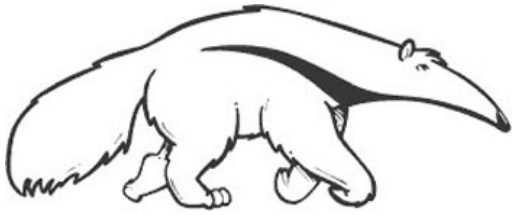

David Scott

(Don’t) Tell Me Another One

- Holds the record of **40 hours and 8 minutes** for the longest stand-up comedy show by an individual, at the Diamond Jo Casino in Dubuque, Iowa, in late April 2013

- He was allowed only a 5 minute break each hour, was required not to repeat any joke within a 4 hour period, and needed to have at least 10 people in the audience at all times.

(Don’t) Tell Me Another One

- One burning question for cognitive science is “when do people give up listening to jokes?”

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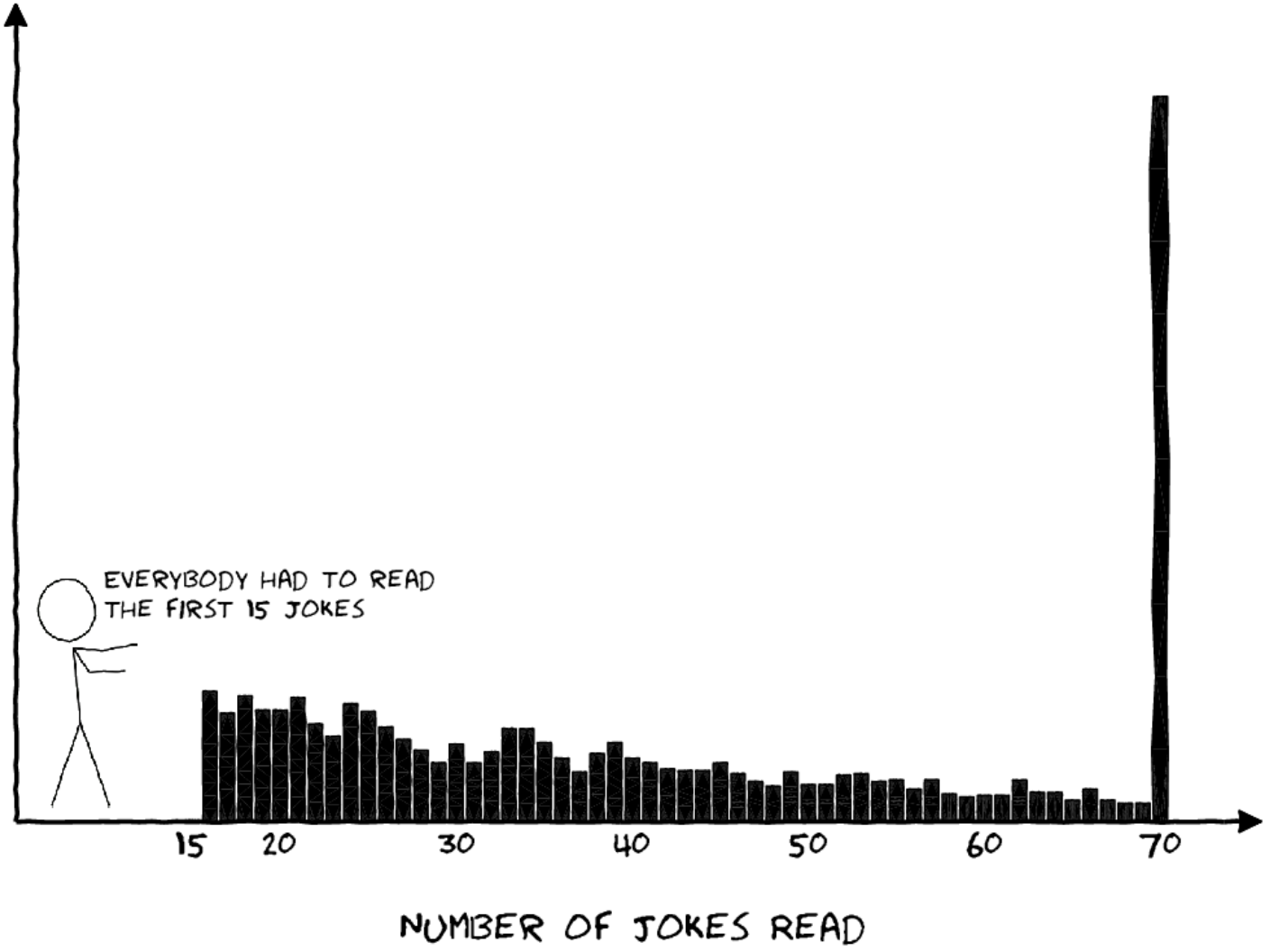
Jester On-Line Joke System

- Distribution of jokes read by 2607 people, at a time when the system had 70 jokes (Goldberg et al, 2001)



A Censored Geometric Model

- Suppose people have some probability of quitting after each joke, and the measurement of this geometric distribution is censored by the on-line joke system only having 70 jokes

$$\theta \sim \text{Uniform}(0, 1)$$

$$\alpha_i \sim 15 + \text{Geometric}(\theta)$$

$$y_i = \begin{cases} \alpha_i & \text{if } \alpha_i \leq 70 \\ 70 & \text{if } \alpha_i > 70 \end{cases}$$

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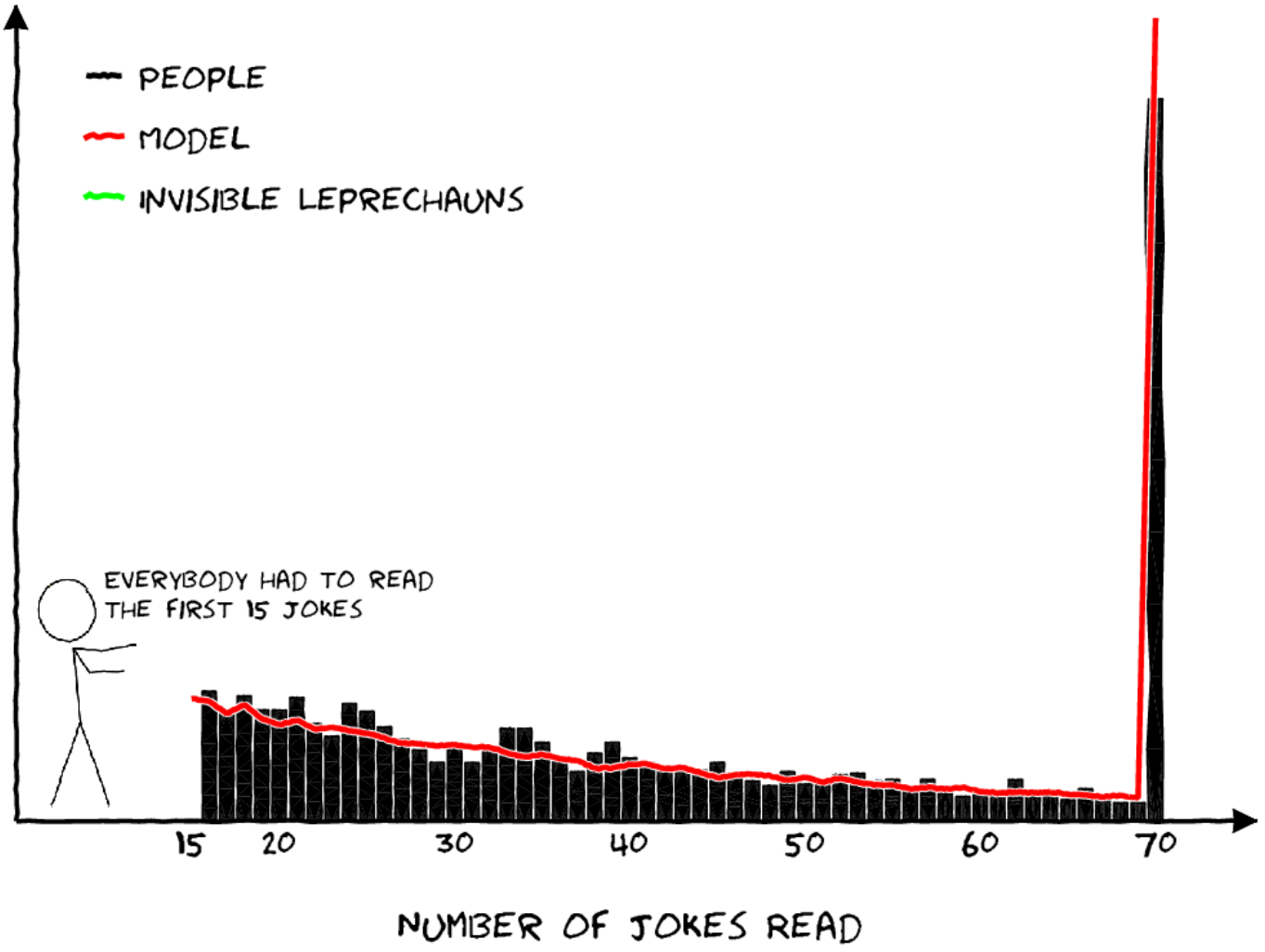
```
model{
  theta ~ dunif(0,1)
  for (i in 1:nPeople){
    alpha[i] ~ dnegbin(theta,1)
    # b is [1,2,...,70]
    y[i] ~ dinterval(alpha[i],b)
  }
}
```

Posterior Predictive Agreement

- This simple model is descriptively adequate, with a termination probability around 3% for each person on each joke



An Extended Model with Individual Differences

- Two sorts of individual differences
  - people can have different termination probabilities
  - some people just always have to read all the jokes

$$\mu \sim \text{Uniform}(0, 1)$$

$$\sigma \sim \text{Uniform}(0, 1)$$

$$\theta_i \sim \text{Gaussian}_{(0,1)}\left(\mu, \frac{1}{\sigma^2}\right)$$

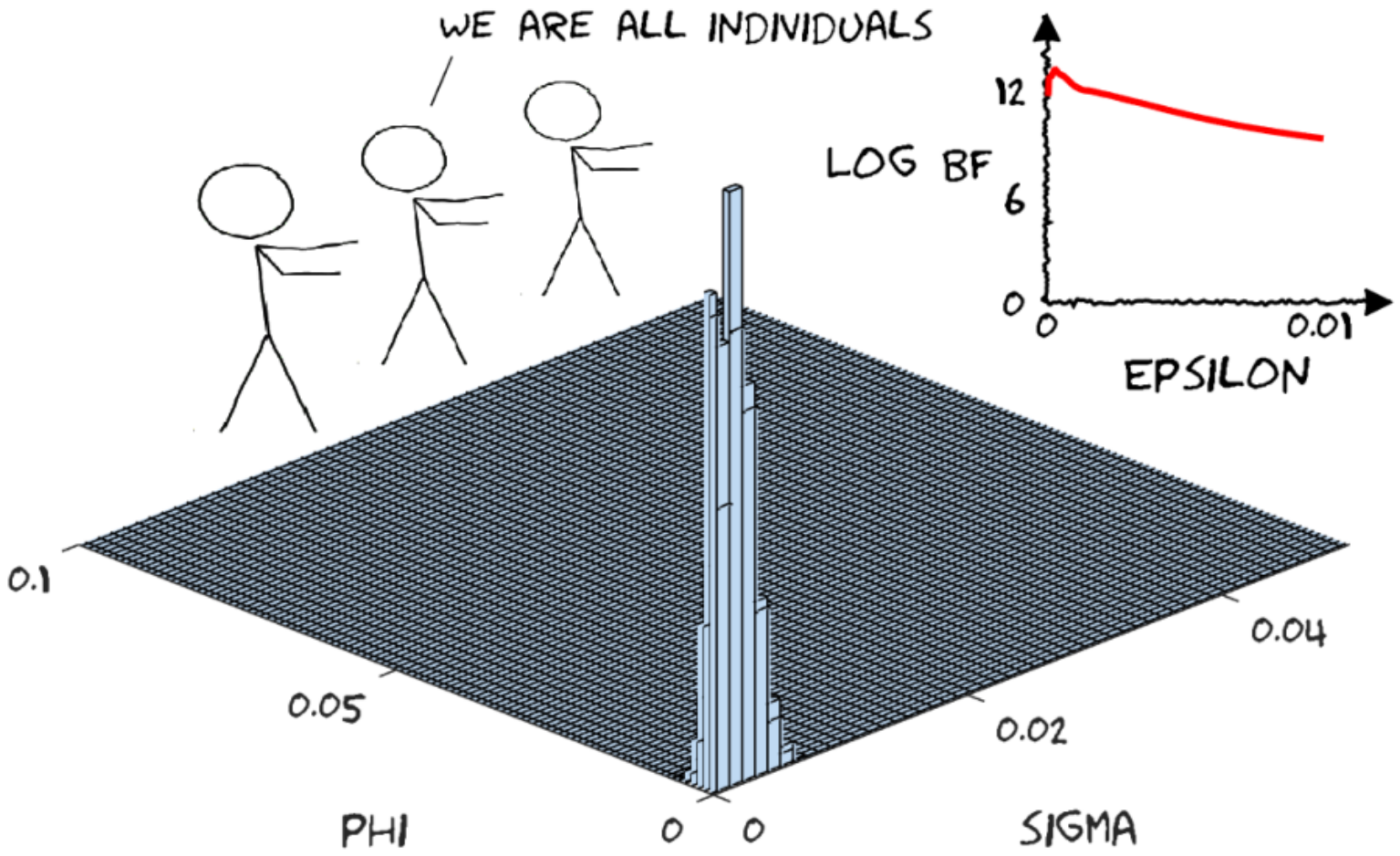
$$\phi \sim \text{Uniform}(0, 1)$$

$$z_i \sim \text{Bernoulli}(\phi)$$

$$y_i \sim \begin{cases} 15 + \text{Geometric}\,(\theta_i) & \text{if } z_i = 0 \text{ and } \alpha_i \leq 70 \\ 70 & \text{if } z_i = 0 \text{ and } \alpha_i > 70 \\ 70 & \text{if } z_i = 1. \end{cases}$$

Are There Individual Differences?

- Bayes factor provides evidence for the original simple model, so these data provide no evidence for the individual differences



Bayesian Statistics and Cognitive Science

Bayesian methods

- Bayesian methods let you infer parameters, evaluate models, and understand and make predictions about data
- Three types of application in psychology

Bayesian methods

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- Three types of application in psychology
  - **Bayes in the head:** Use Bayes as a theoretical metaphor, assuming that when people make inferences they apply Bayesian methods (at some level)

Josh Tenenbaum

Tom Griffiths

Mark Steyvers

Charles Kemp

Bayesian methods

- Bayesian methods let you infer parameters, evaluate models, and understand and make predictions about data
- Three types of application in psychology
  - **Bayes in the head**
  - **Bayes for data analysis:** Instead of using frequentist estimation, confidence intervals, null hypothesis testing, and so on, use Bayesian inference to analyze data

EJ Wagenmakers

Jeff Rouder

Joachim Vandekerckhove

Richard Morey

John Kruschke

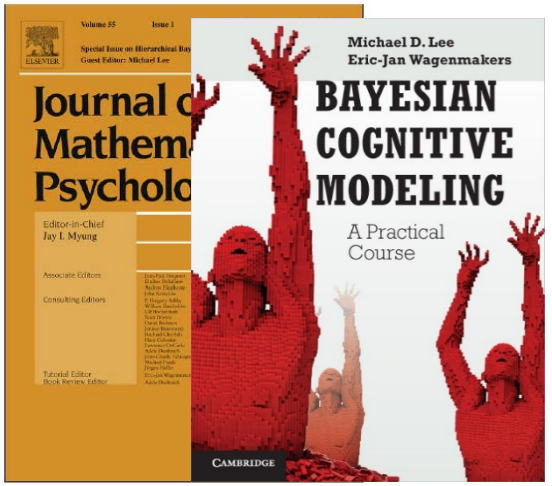
Bayesian methods

- Bayesian methods let you infer parameters, evaluate models, and understand and make predictions about data
- Three types of application in psychology
  - **Bayes in the head**
  - **Bayes for data analysis**
  - **Bayes for cognitive modeling**: Use Bayesian inference to relate models of psychological processes to behavioral data

EJ Wagenmakers

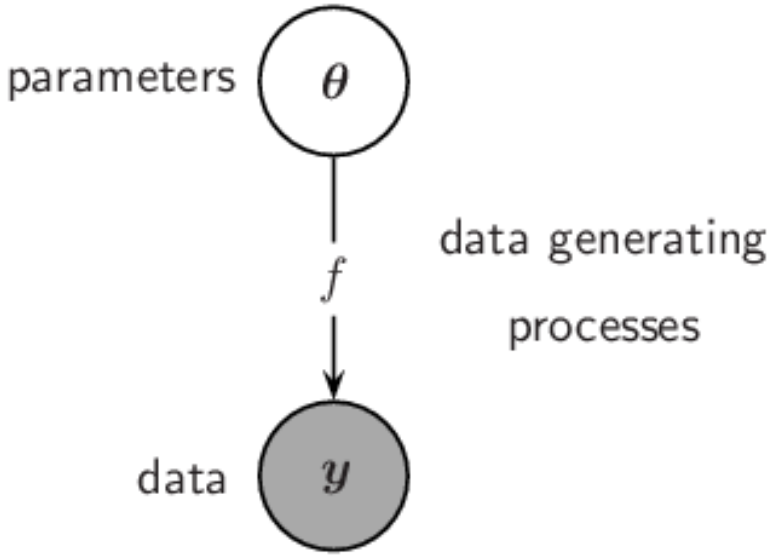
Jeff Rouder

Wolf Vanpaemel



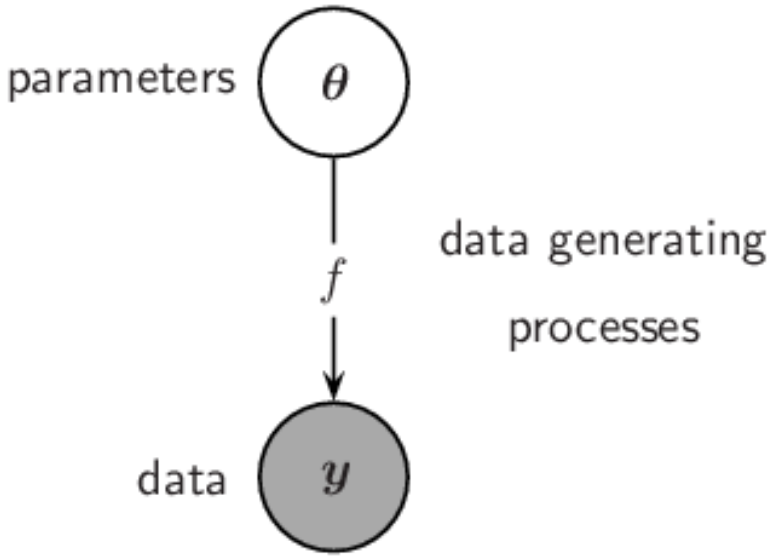
Cognitive Models as Generative Statistical Models

- Most cognitive models can be thought of as data generating processes, combining
  - Psychological processes, formalized by the likelihood
  - Psychological variables, formalized by parameters



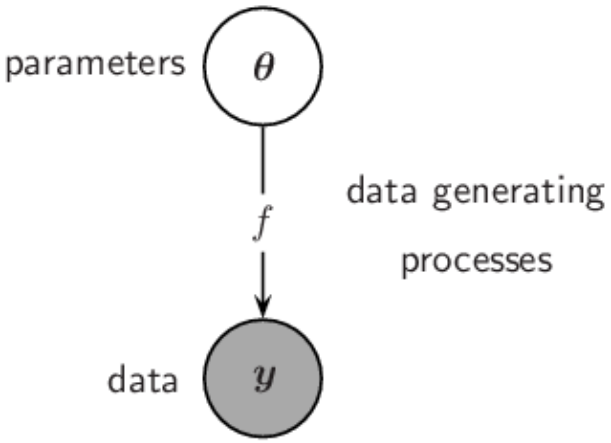
Attraction of Bayesian methods

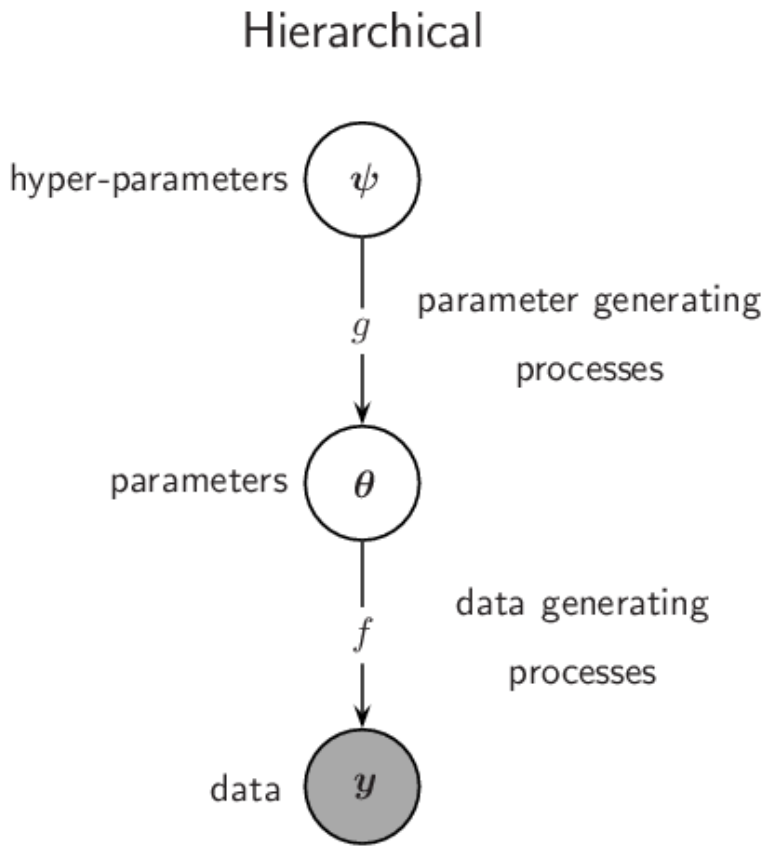
- Beyond the conceptual coherence and completeness, the great advantage of Bayesian methods is they allow cognitive that are more complicated than the standard one to be considered



Hierarchical modeling

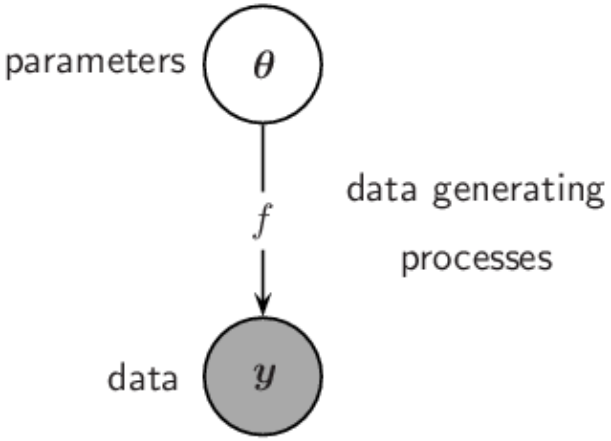
- Hierarchical models extend the standard approach by including a modeling account of how the basic model parameters themselves are generated

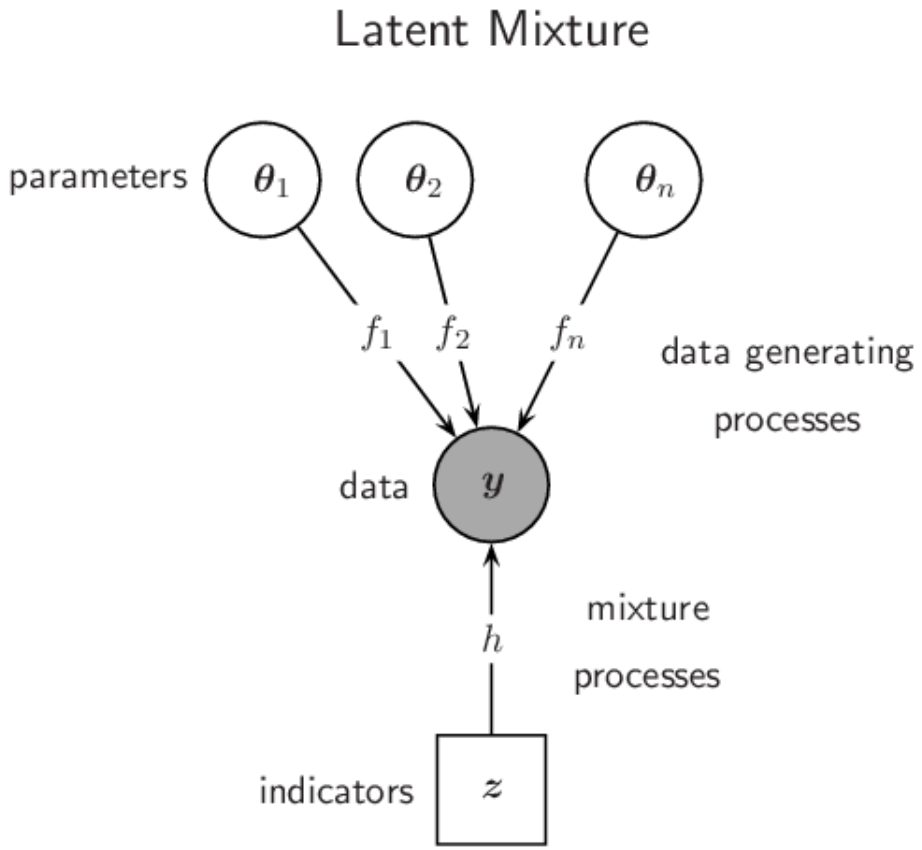




Latent-mixture modeling

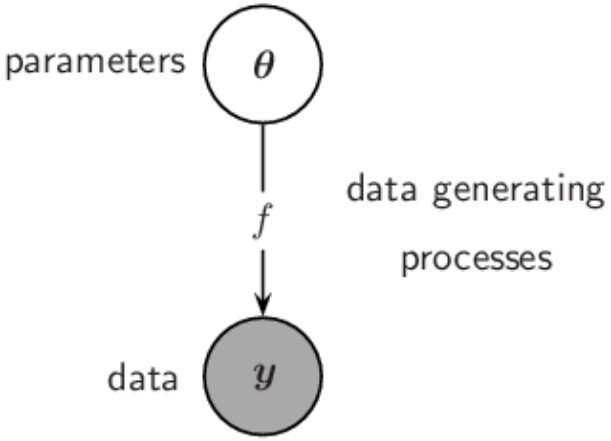
- Latent-mixture models extend the standard approach by allowing behavioral data to be generated as a mixture of multiple different processes and controlling parameters

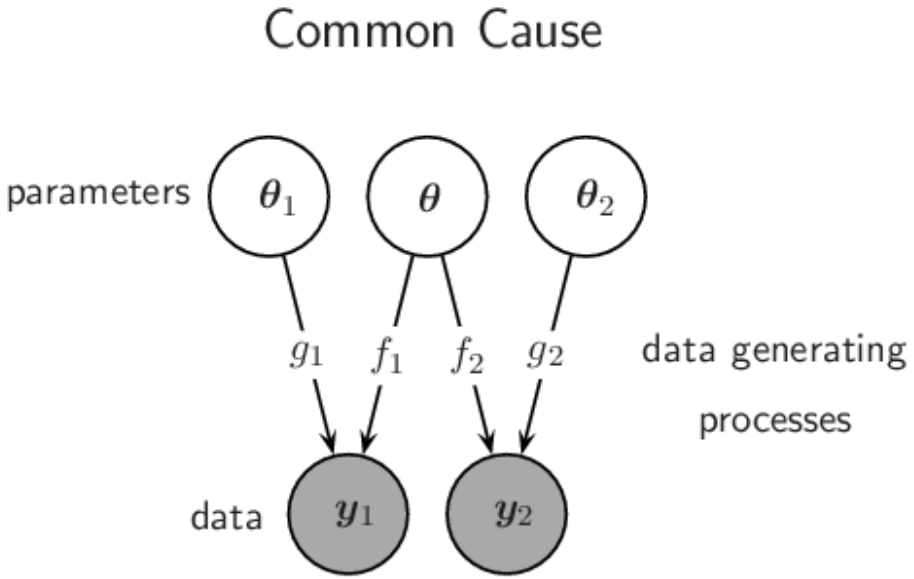




Common-cause modeling

- Common-cause models extend the standard approach by allowing the same psychological variables to influence multiple sorts of observed behavior





Wisdom of the Crowd for Ranking Data

RANK	SITE	MONTHLY PEOPLE
1	google.com	235,702,560
2	youtube.com	200,078,640
3	facebook.com	142,452,240
4	msn.com	122,279,512
5	yahoo.com	115,347,760
6	bing.com	104,640,000
7	amazon.com	101,915,368
37	foxnews.com	22,965,512
38	blogger.com	22,904,332
39	Hidden profile	—
40	ranker.com	21,974,350
41	Hidden profile	—
42	answers.com	20,358,508
43	craigslist.org	20,114,412

Ultimate List | originally by Ranker Community | tags | 18,327 votes | 2,258 voters | 34,565

The Most Anticipated 2013 Films

List Criteria: Only films with a release date in 2013.

List of the most eagerly anticipated movies opening in 2013. Before the year gets going, it' people are most excited about seeing in the coming year. Will these movies live up to exp dominate people's thinking about 2013 films, with the most conventionally exciting titles ev on a list like this in advance can potentially answer these questions down the road. (Don't **Anticipated Films of 2012 here!**)

2013 will continue many of the trends set in the preceding years. Remakes remain a stapl

SORRY, VOTING AND RERANKING FOR THIS LIST HAS BEEN CLOSED.

SEE LIST RANKED BY | REGION | AGE | MEN | WOMEN | 26 RERANKS

THE LIST | OPTIONS | Actors | bonnie4arq

1	681	252	*the people's #1* Man of Steel Amy Adams, Diane Lane, Ke	The Hobbit: The Desolation of Smaug Cate Blanchett, Evangeline Lilly, Orlando Bloom,
2	640	225	*most listed* The Hobbit: Th Cate Blanchett, Evangeline L	The Lone Ranger Johnny Depp, Helena Bonham Carter, Armie
3	873	191	*ranked low on reranks* Iron Man 3 Gwyneth Paltrow, Robert Do	The Great Gatsby Leonardo DiCaprio, Isla Fisher, Carey Mulligan,
4	606	202	Thor: The Darl Natalie Portman, Kat Dennin	Now You See Me Morgan Freeman, Isla Fisher, Michael Caine,
5	692	211	Star Trek Into Zoe Saldana, Benedict Cumb	Percy Jackson: Sea of Monsters Alexandra Daddario, Logan Lerman, Jake Abel,

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bonnie4arq

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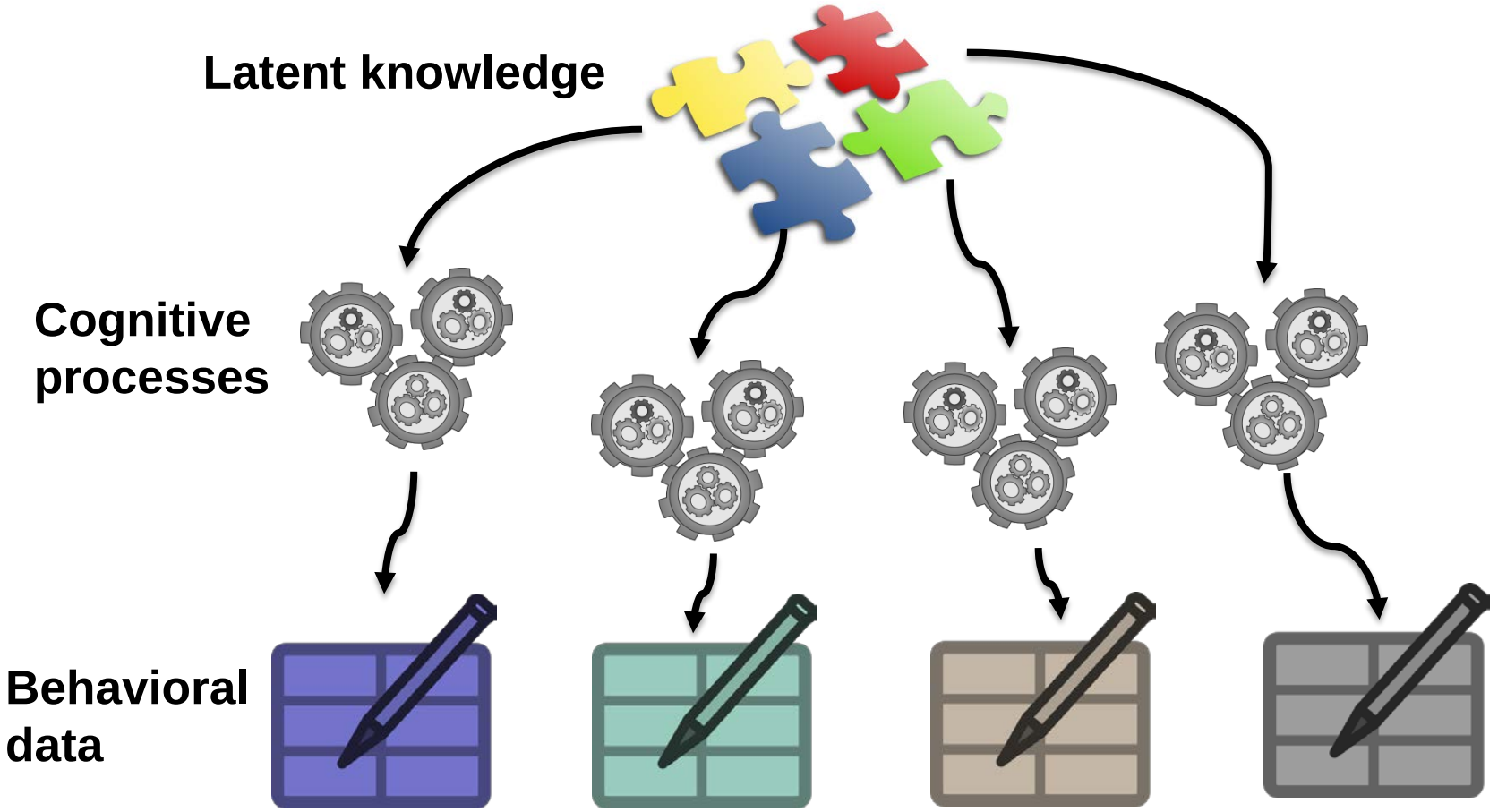
5	Percy Jackson: Sea of Monsters Alexandra Daddario, Logan Lerman, Jake Abel,

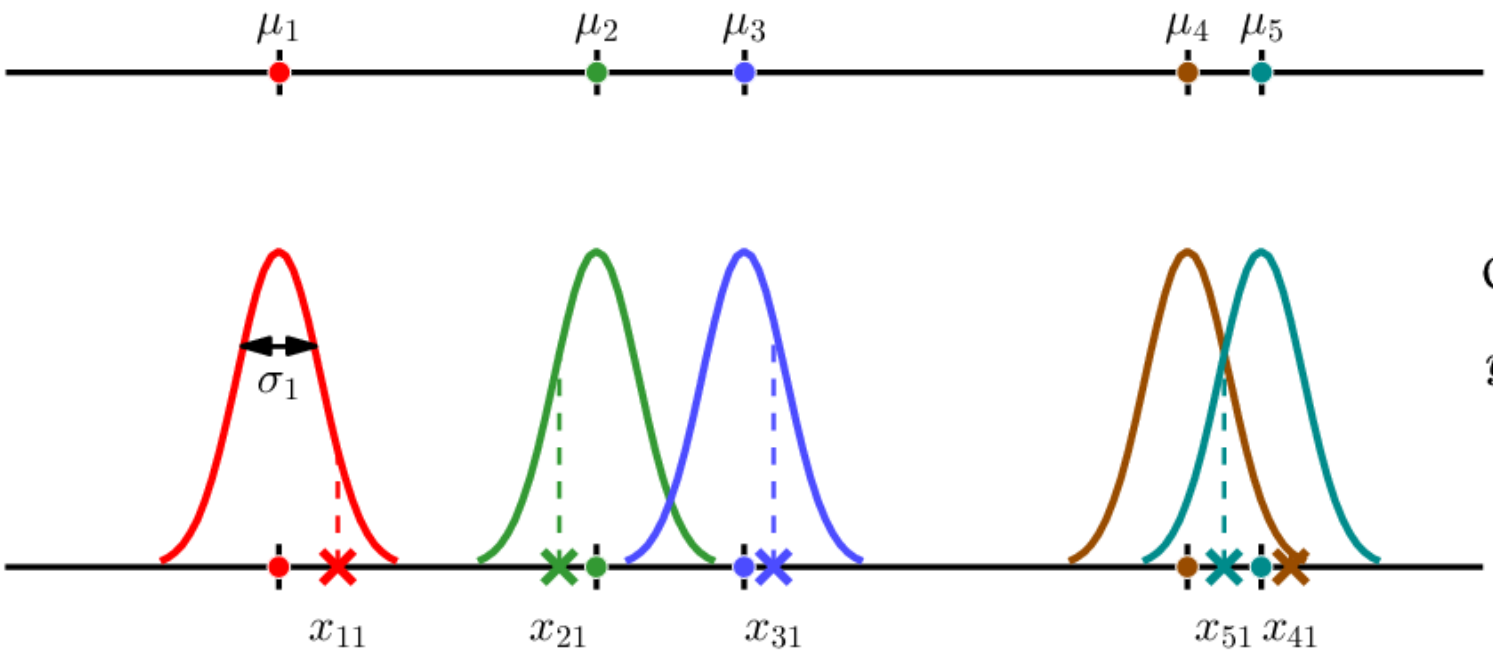
Ranking

Generative Cognitive Modeling Approach

- Use cognitive models of the processes that generate data, and the individual differences across people, as the bridge between the available data and the latent group knowledge



Thurstone (1927) Model for Partial Lists



Observed Ranking

$\boldsymbol{y}_1 = (1, 2, 3, 5)$

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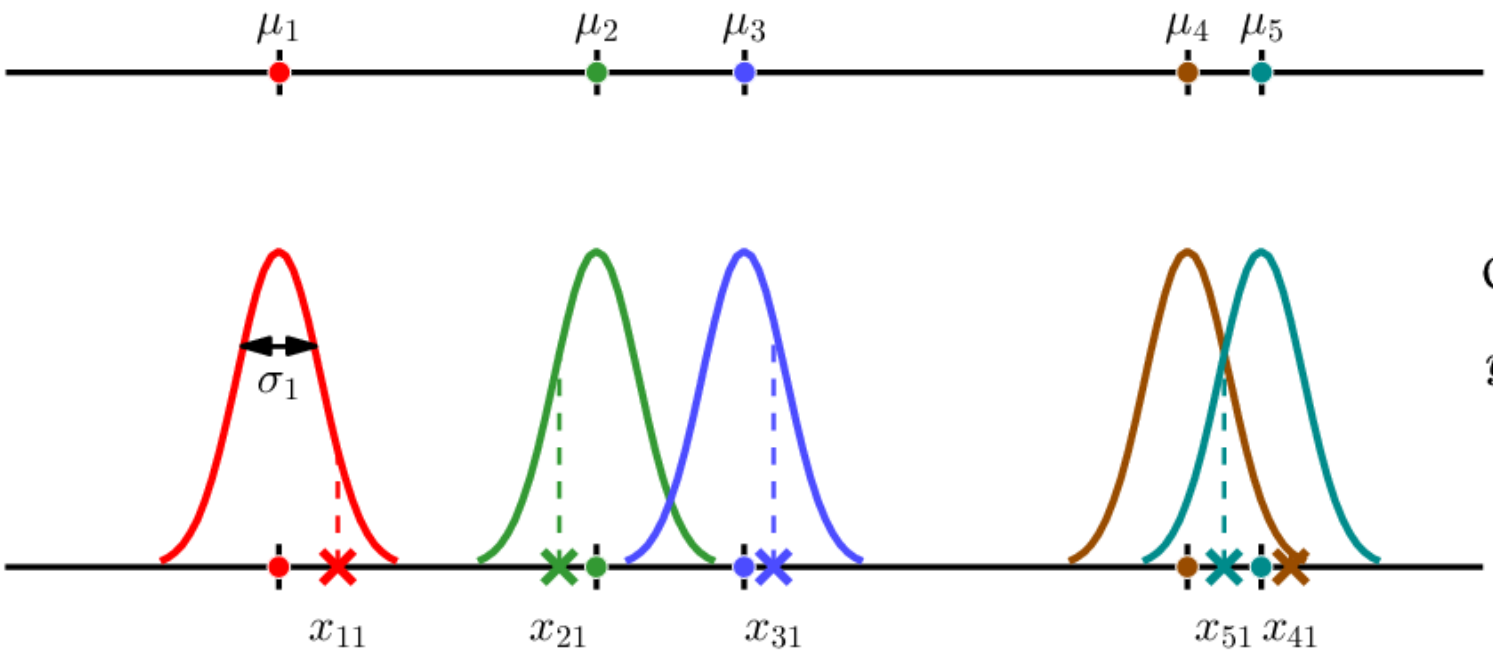

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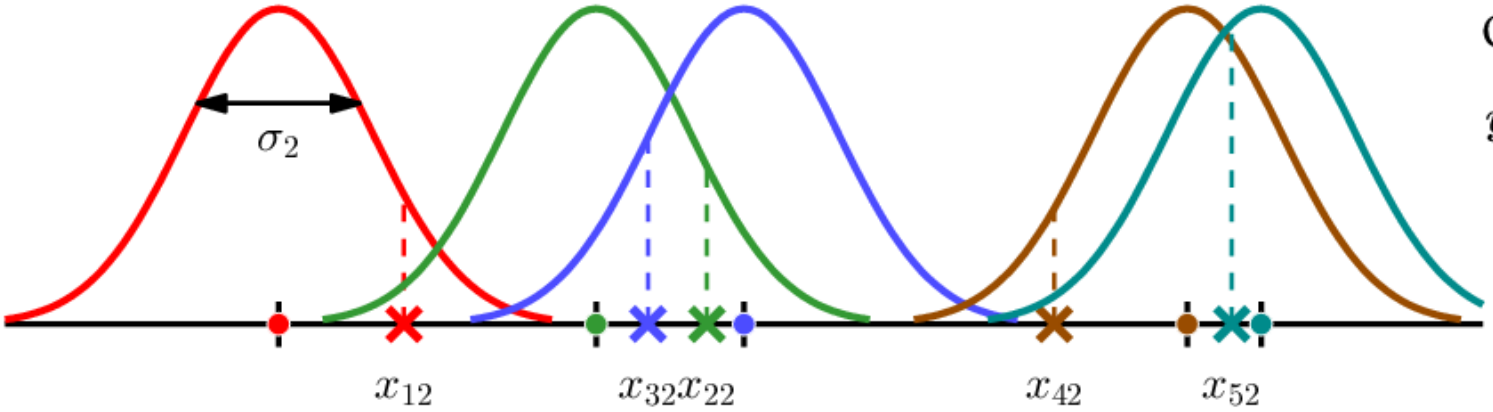
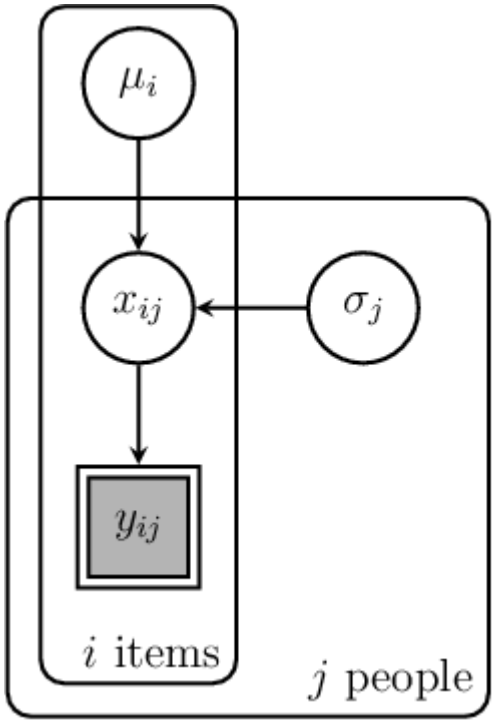


Observed Ranking

$\boldsymbol{y}_2 = (1, 3, 2)$

Graphical Model

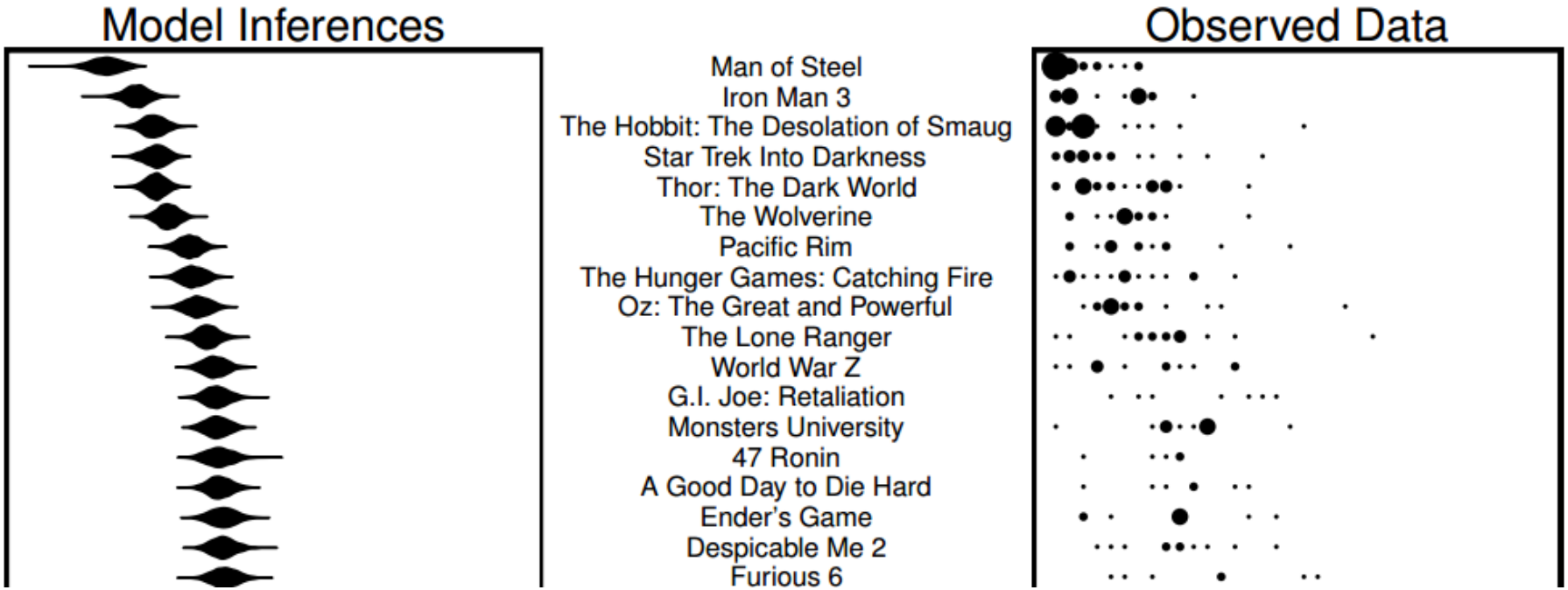
- Implement the Thurstonian model as a graphical model in JAGS, inferring two sorts of psychological parameters
  - latent item locations
  - expertise of each person



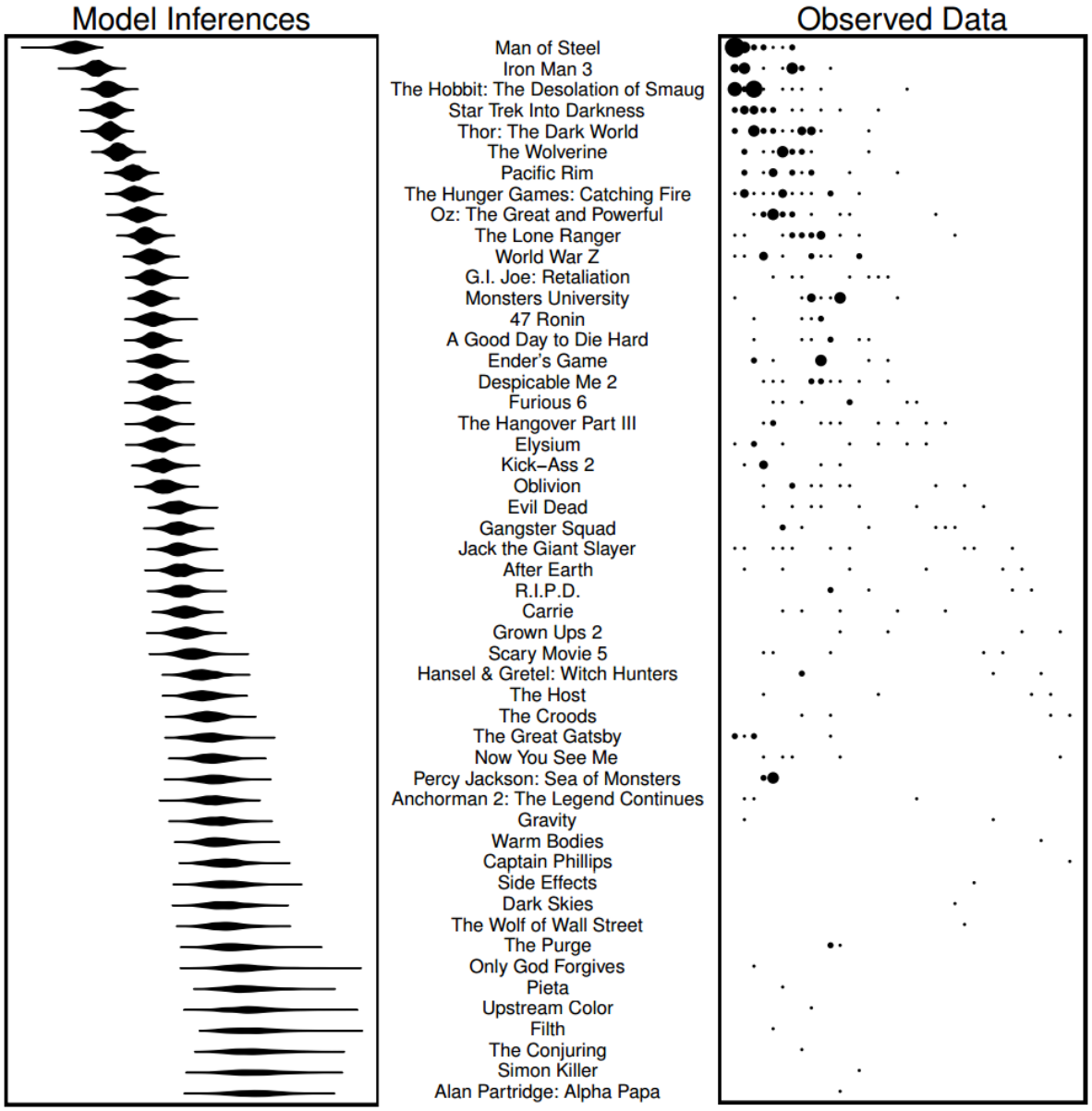
$$
\begin{aligned}
\mu_i &\sim \text{Uniform}(0, 1) \\
\sigma_j &\sim \text{Uniform}(0, 1) \\
x_{ij} &\sim \text{Gaussian}(\mu_i, 1/\sigma_j^2) \\
y_{ij} &= \text{Rank}(x_{ij})
\end{aligned}
$$

Movie Inferences

- Order movies in terms of expected posterior of their inferred item location

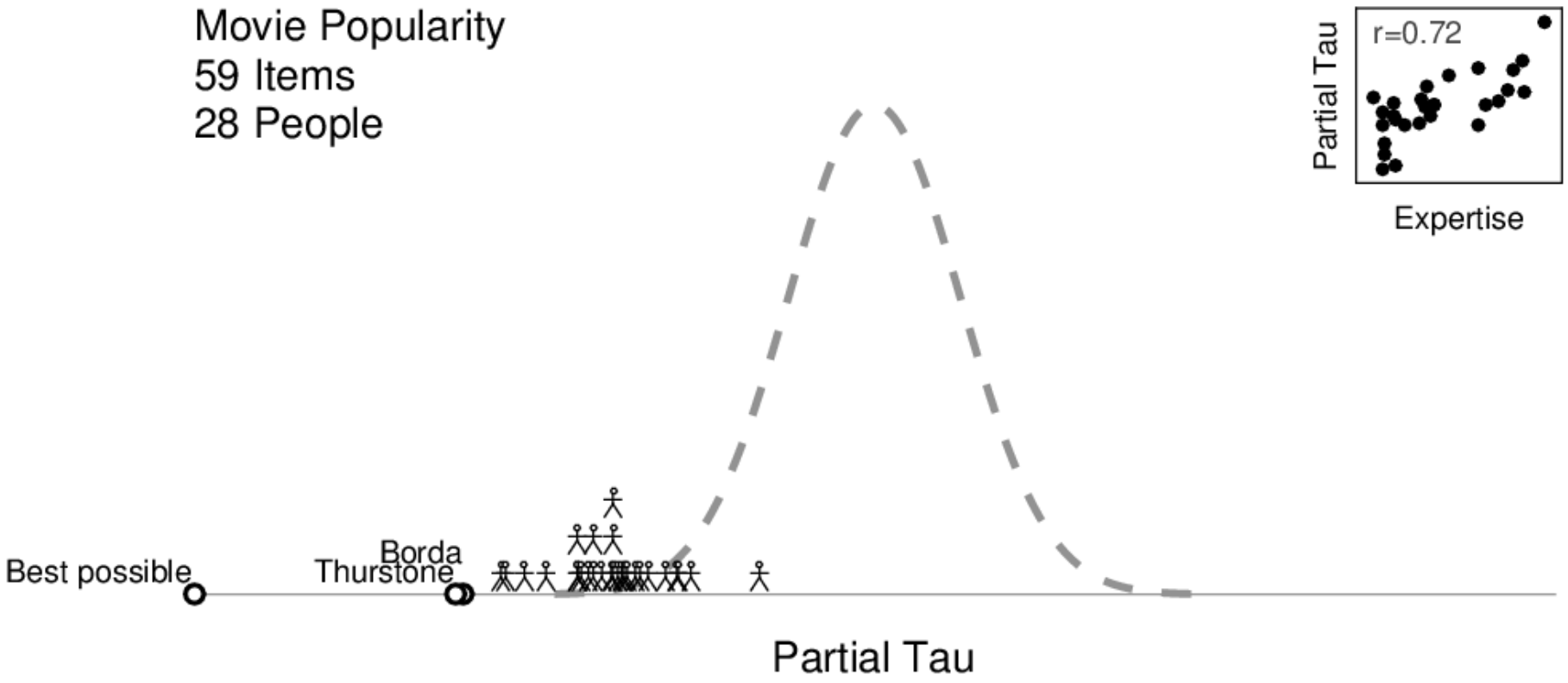


Movie Inferences



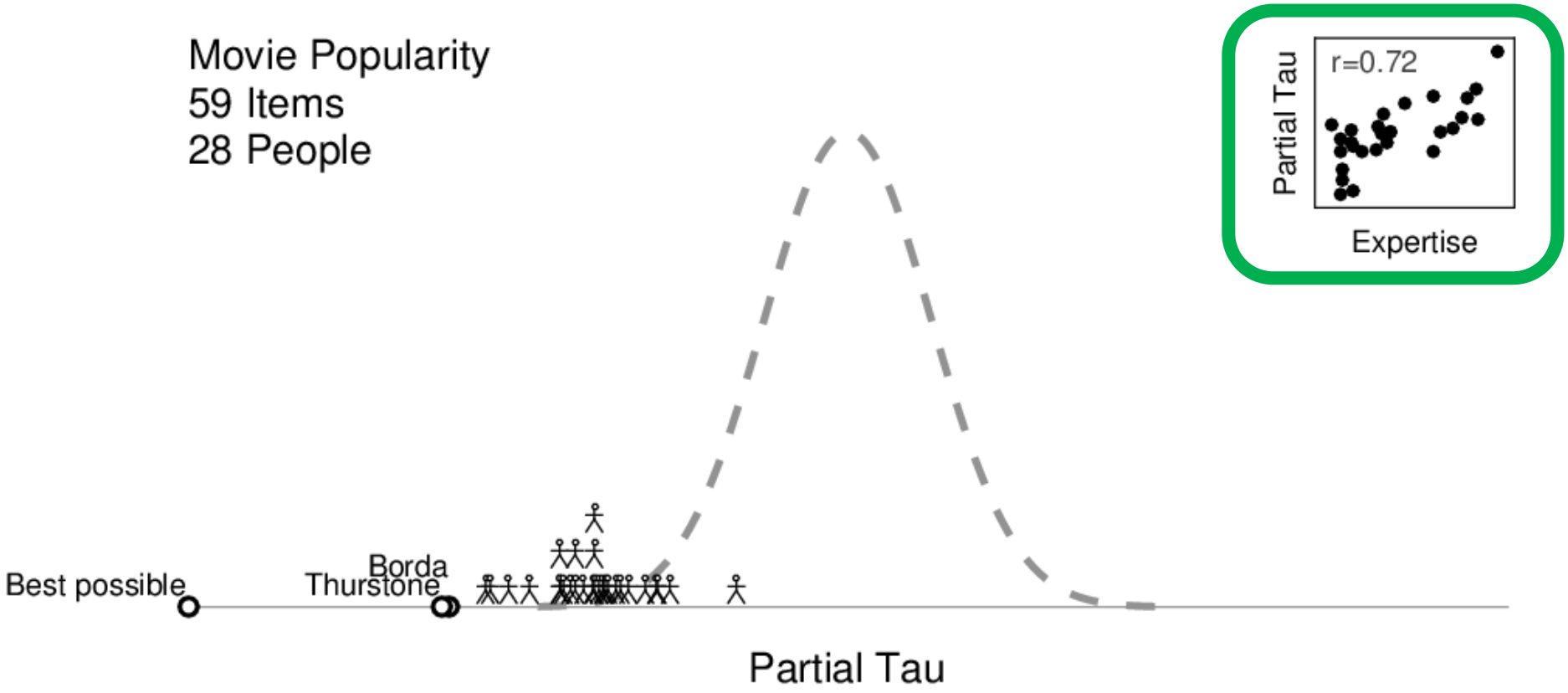
Movie Performance

- We used ground truth box office takings from imdb.com, and partial tau to measure how well individuals, the model, and the Borda count statistical method performed



Movie Performance

- Both the model and Borda count make very good predictions, and are better than even the best individual
- The model also provides a good prediction of relative expertise



Vote List | by Ranker Soccer ★ RANKER STAFF | tags | 15,784 votes | 2,083 voters | 43,207 views

Who Will Win the 2014 World Cup

List Criteria: Only teams participating in the 2014 World Cup

A list of what country will win the 2014 World Cup. It is the most watched event in the wor
top 32 countries from across the globe will descend upon Brazil to compete for what's co
trophies in the world. The best footballing countries in the world aim to leave Brazil with th
winners, bringing prestige and honor to their respective countries, and coming home as c
it won't be going anywhere).

Just ask Spain what it meant when their players took home the 2010 trophy, the first in its

SORRY, VOTING AND RERANKING FOR THIS LIST HAS BEEN CLOSED.

SEE LIST RANKED BY | REGION | AGE | MEN | WOMEN | 19 RERANKS ✕

THE LIST | OPTIONS | Capital ▼ | Scandinavia14 ▼

1	740	468	Germany Berlin	Germany Berlin
2	489	389	Netherlands Amsterdam	Brazil Brasília
3	483	502	Brazil Brasília	Spain Madrid
4	419	461	Argentina Buenos Aires	Russia Moscow
5	227	357	France Paris	Portugal Lisbon

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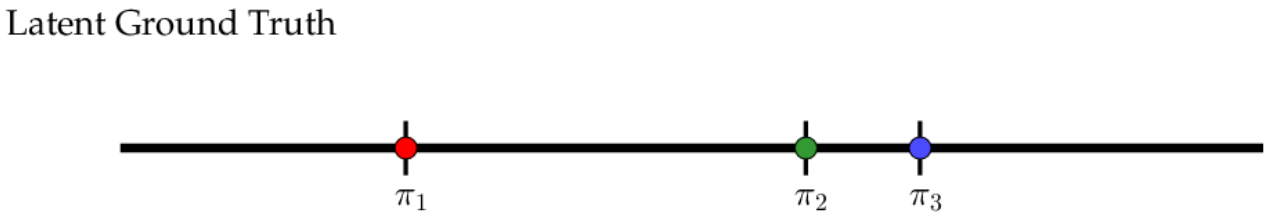
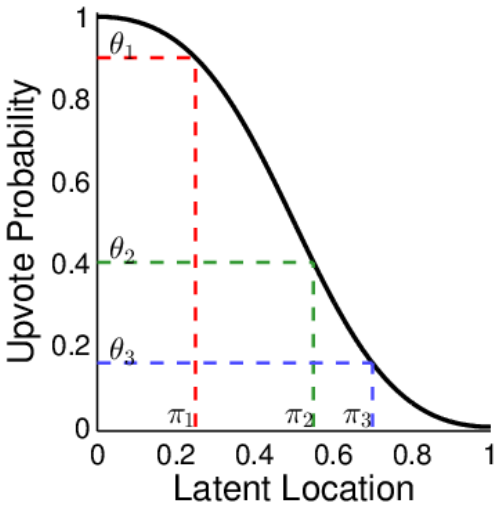
Voting

Modeling Ranking and Voting Data

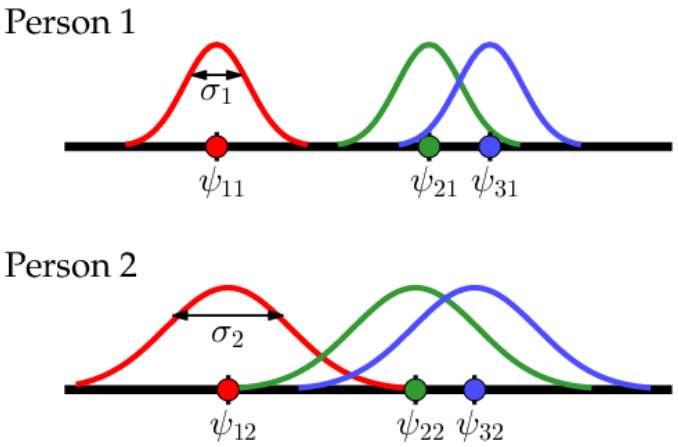
- Extend the Thurstone model of rank data generation to include a transfer function mapping latent item location to up-voting



Voting Data



Ranking Data

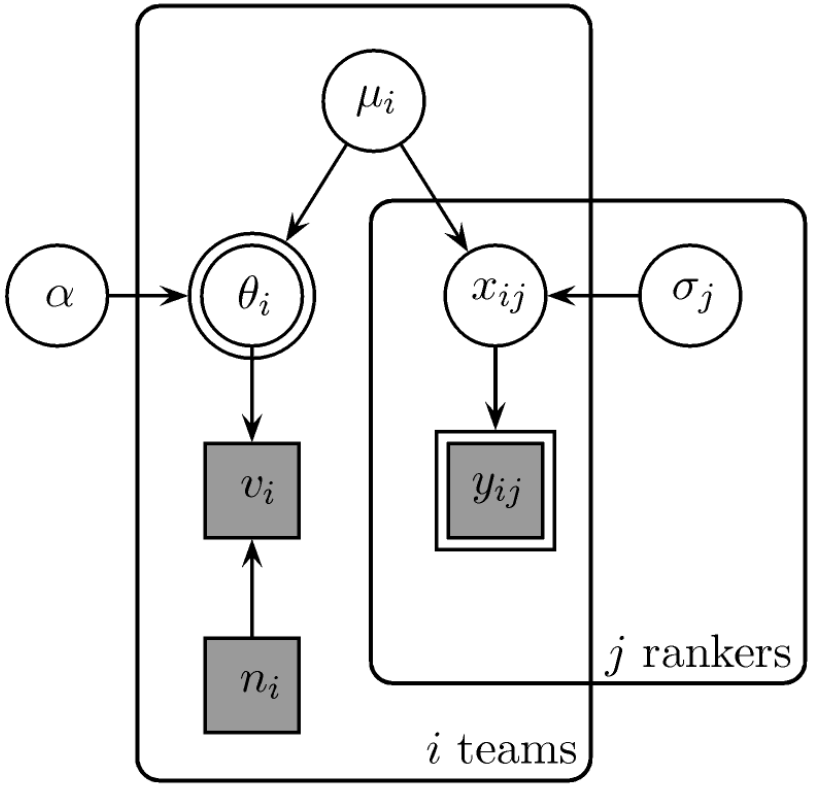


Bracket Prior Information

- The bracket for the World Cup is highly relevant prior information, which is natural for Bayesian analysis

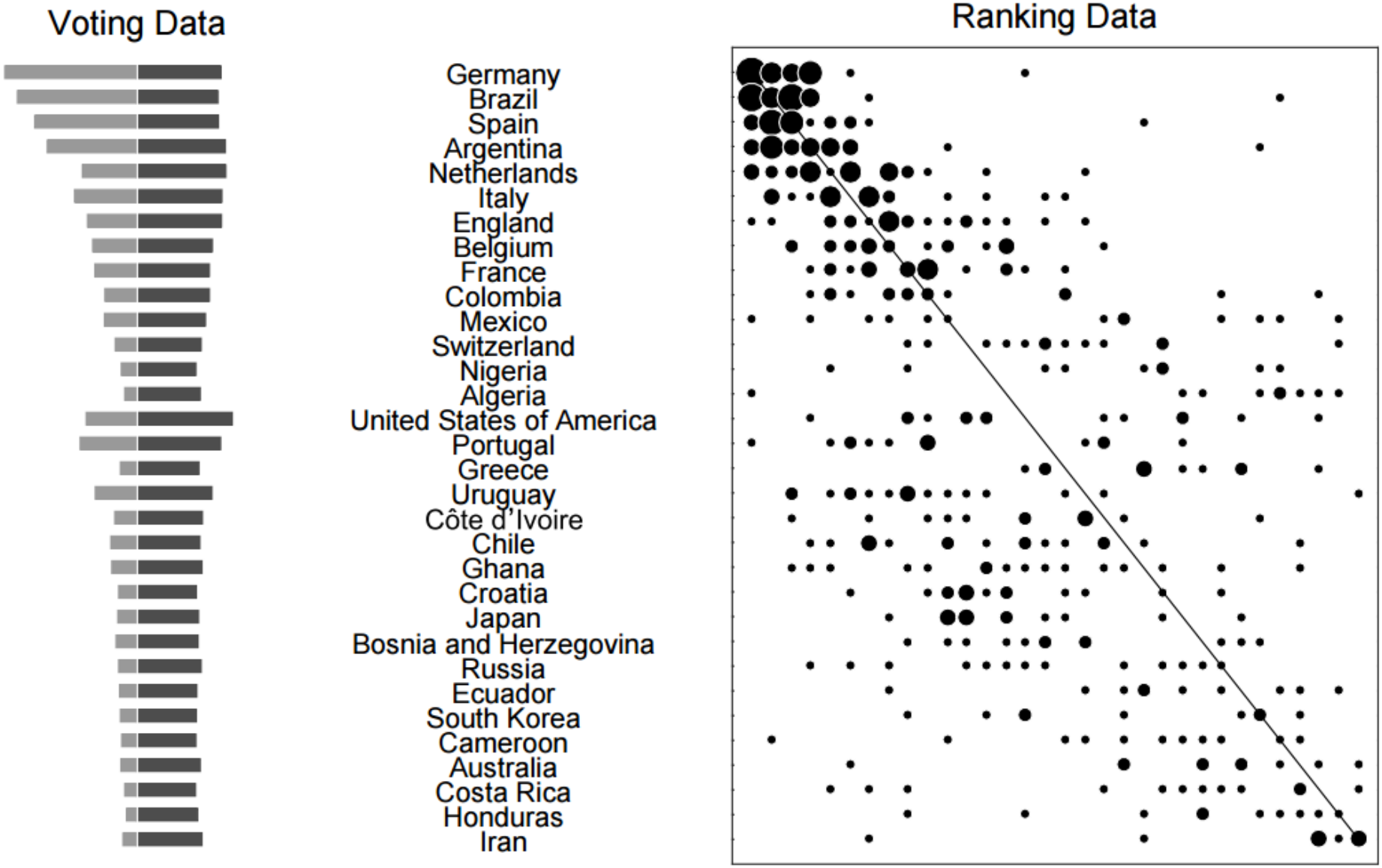
Combining All Relevant Information

- Graphical model combines ranking data, voting data, and bracket structure prior information



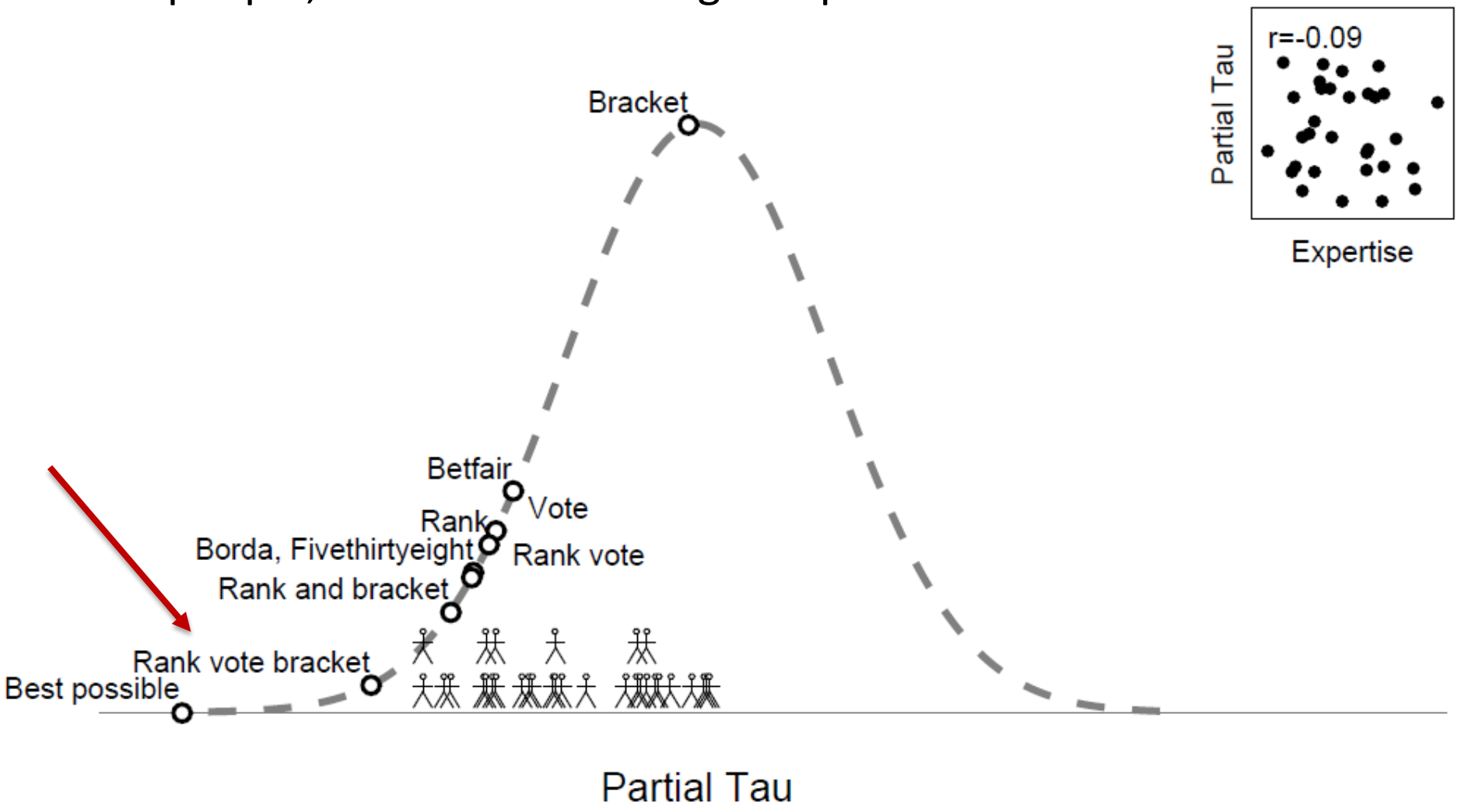
$$
\begin{aligned}
\mu_i &\overset{\mathcal{B}}{\sim} \text{Gaussian}(0, 0.001) \\
\sigma_j &\sim \text{Uniform}(0, 20) \\
x_{ij} &\sim \text{Gaussian}(\mu_i, \sigma_j^{-2}) \\
y_{ij} &\leftarrow \text{Rank}\,(x_{ij}) \\
\alpha &\sim \text{Gaussian}(0, 0.01) \\
\theta_i &\leftarrow 1/\left(1 + \exp\left\{-\alpha\mu_i\right\}\right) \\
v_i &\sim \text{Binomial}(\theta_i, n_i)
\end{aligned}
$$

Model World Cup Predictions



World Cup Performance

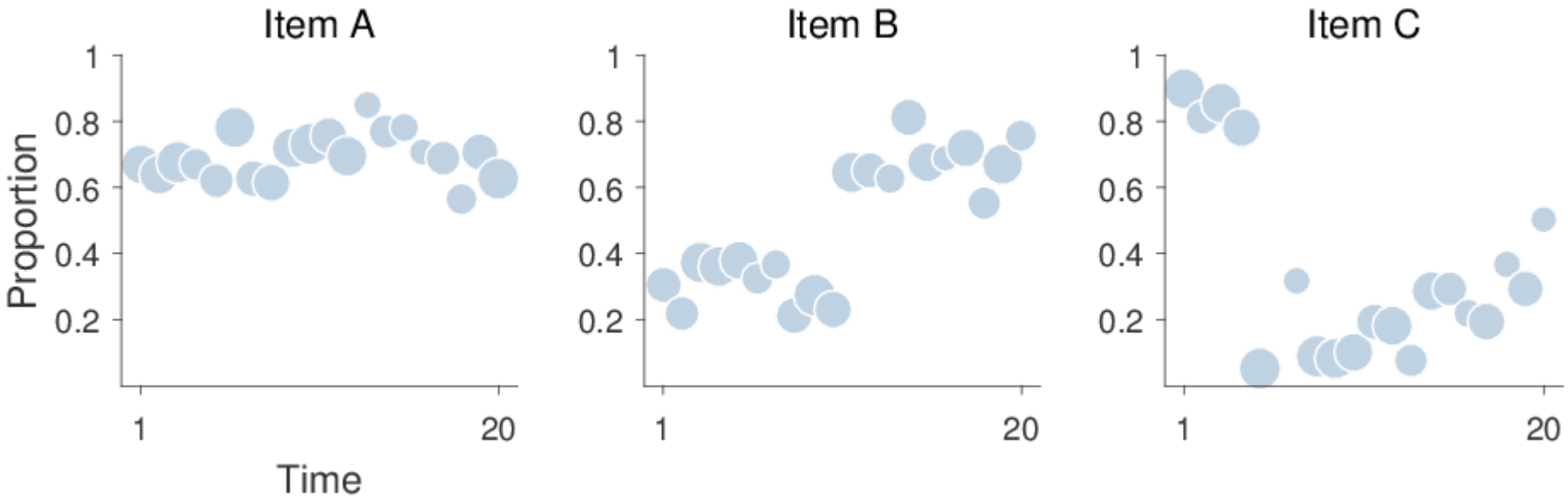
- Combining all the information makes a better prediction than all people, and various betting and prediction sites



Detecting Step Changes in Cognition

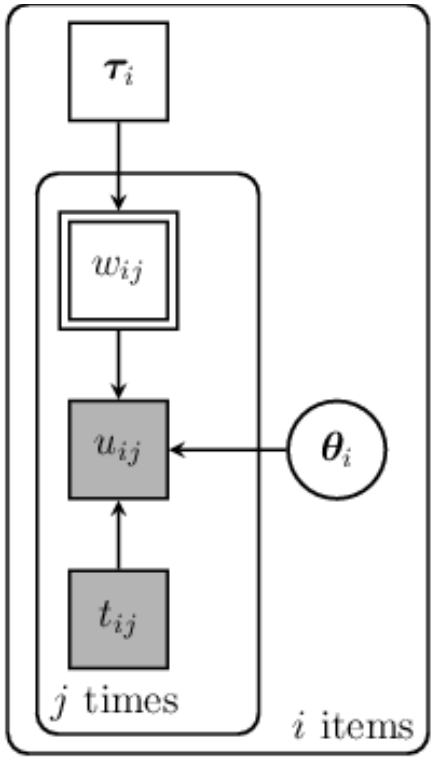
Detecting Step Change

- Lots of cognitive phenomena and observed behaviors change suddenly
  - Large statistical literature on inferring how many changes there are, and where they occur (Barry & Hartigan 1993, Chib 1998, Fearnhead 2006, Adams and Mackay 2007)



A Graphical Model of Detecting Change

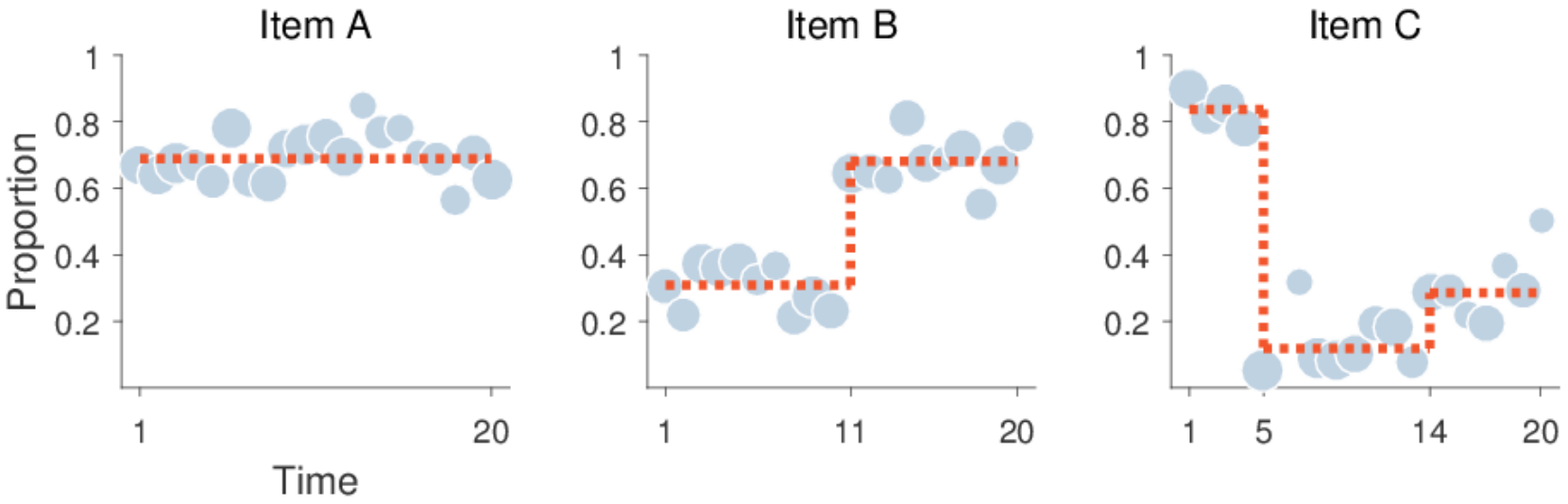
- Key is the spike-and-slab prior on the (ordered) change points
  - half the prior mass to “no change”
  - other half distributed equally over meaningful possibilities

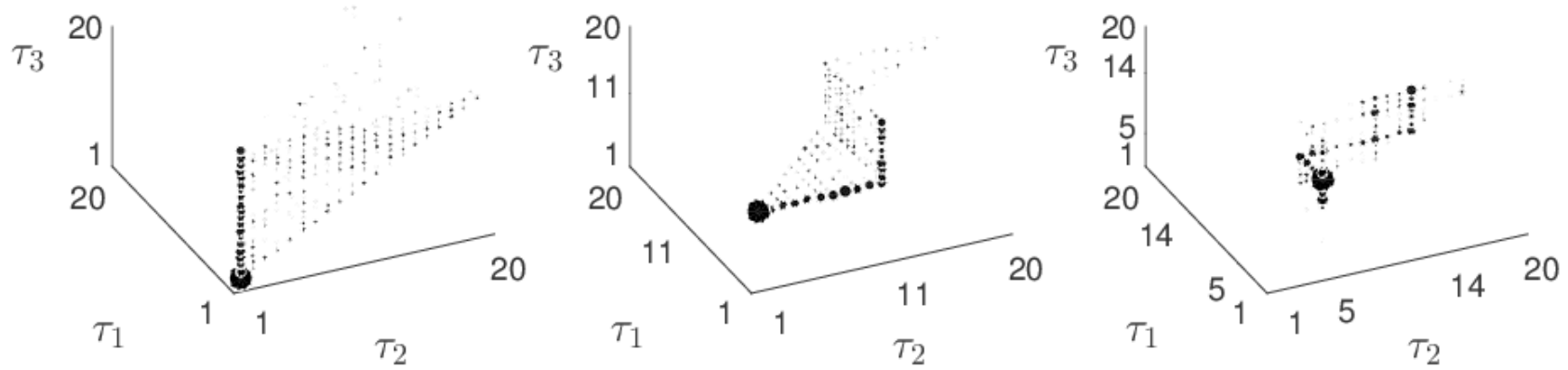


$$
\begin{aligned}
\theta_{ik} &\sim \text{Uniform}(0,1) \\
\tau_{ik} &\sim \text{Categorical}\Big(\overbrace{\frac{n}{2n-1}, \frac{1}{2n-1}, \ldots, \frac{1}{2n-1}}^{n \text{ times}}\Big); \tau_1 \leq \ldots \leq \tau_\gamma \\
w_{ij} &= \textstyle\sum_k \mathcal{I}\left(j \geq \tau_{ik}\right) \\
u_{ij} &\sim \text{Binomial}\left(\theta_{i,w_{ij}}, t_{ij}\right)
\end{aligned}
$$

Demonstration With Toy Data

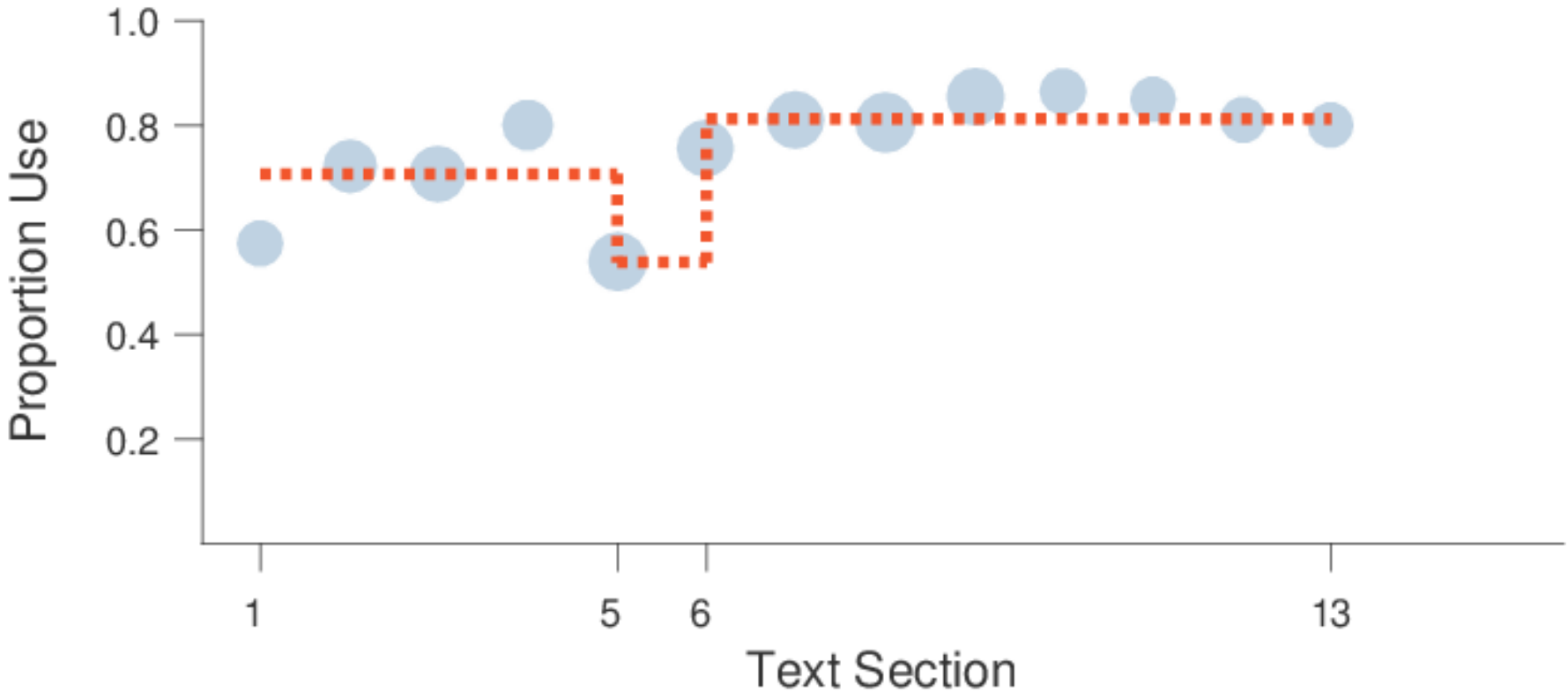
- Take the posterior mode as point estimate of change points, and infer rates conditional on those points





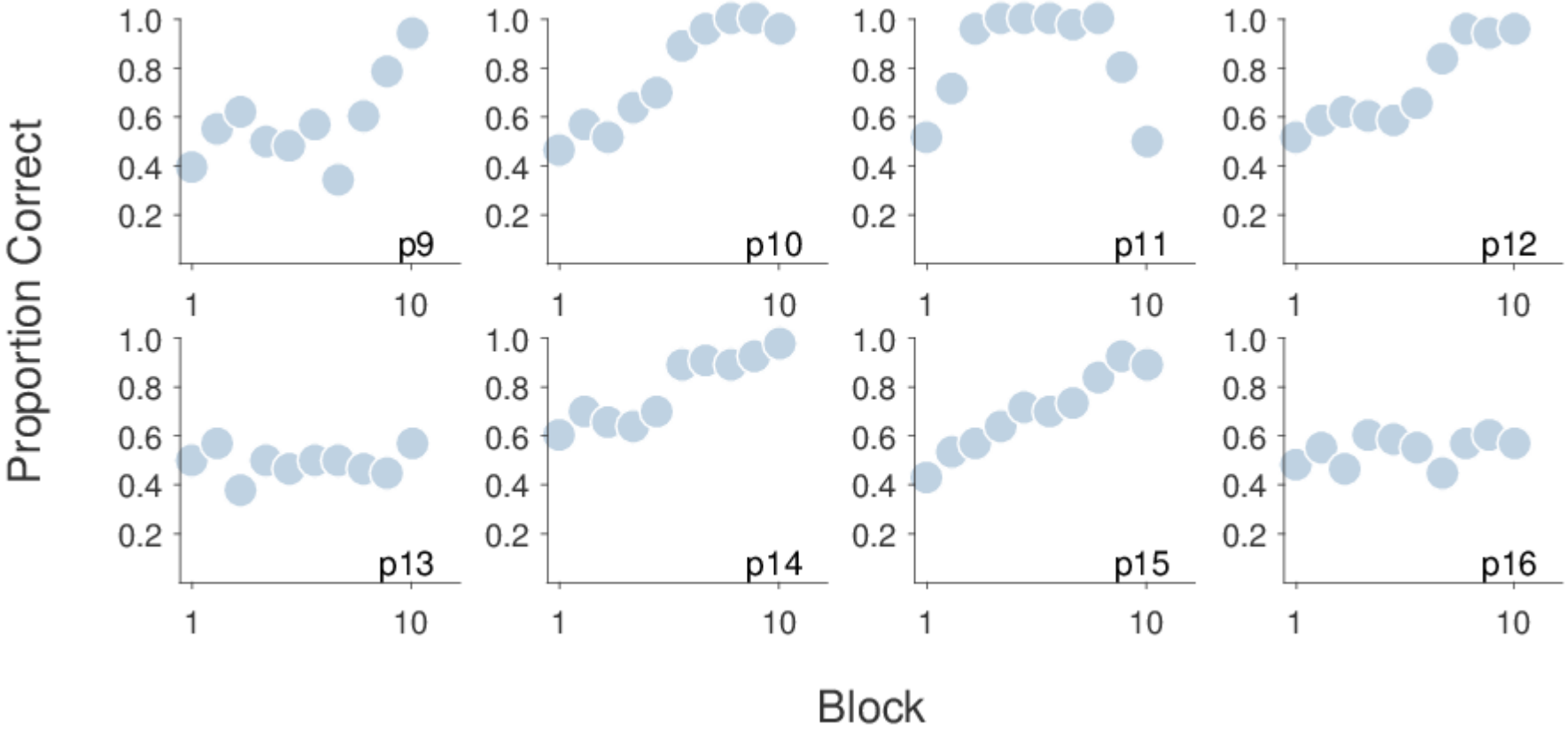
Lindisfarne Scribes Problem

- Looks at the proportion of use of forms like “drives” vs “driveth” in 13 ordered gospels, as a basis for inferring changes in the scribes
- Our inferences match the analytic ones of Smith (1975)



Category Learning Performance

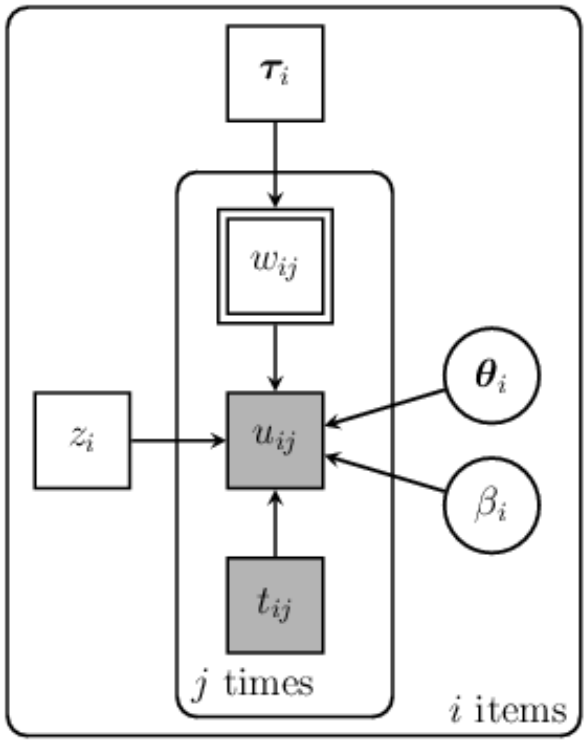
- Cognitive science is interested in whether people learn categories
  - Incrementally, by fine-tuning associations
  - Suddenly, by swapping in and out different candidate rules



Data from Smith and Minda (1998)

Latent-Mixture Model of Category Learning Performance

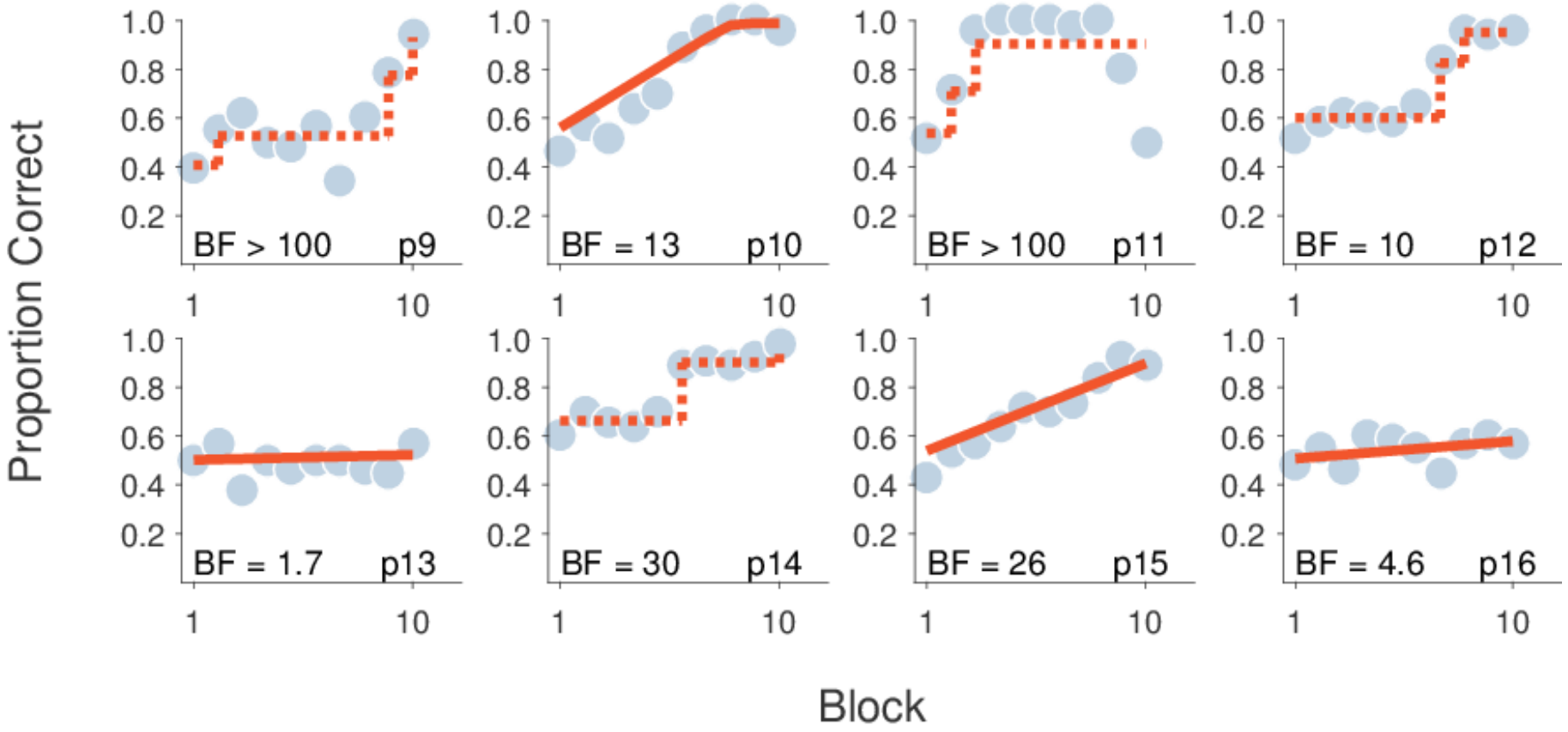
- Allow for two different learning models
  - a linear increase starting at chance and with a threshold at perfect accuracy
  - some number of sudden upward jumps, using change point model



$$
\begin{aligned}
\theta_{ik} &\sim \text{Uniform}(0, 1) \\
\beta_i &\sim \text{Uniform}(0, 0.5) \\
z_i &\sim \text{Bernoulli}(0.5) \\
\tau_{ik} &\sim \text{Categorical}(\overbrace{\frac{n}{2n-1}, \frac{1}{2n-1}, \ldots, \frac{1}{2n-1}}^{n \text{ times}}); \tau_1 \leq \ldots \leq \tau_\gamma \\
w_{ij} &= \textstyle\sum_k \mathcal{I}\left(j \geq \tau_{ik}\right) \\
u_{ij} &\sim \begin{cases} \text{Binomial}\left(\theta_{i,w_{ij}}, t_{ij}\right) & \text{if } z = 1 \\ \text{Binomial}\left(\max\left[1, \beta_i \times j + 0.5\right], t_{ij}\right) & \text{if } z = 0 \end{cases}
\end{aligned}
$$

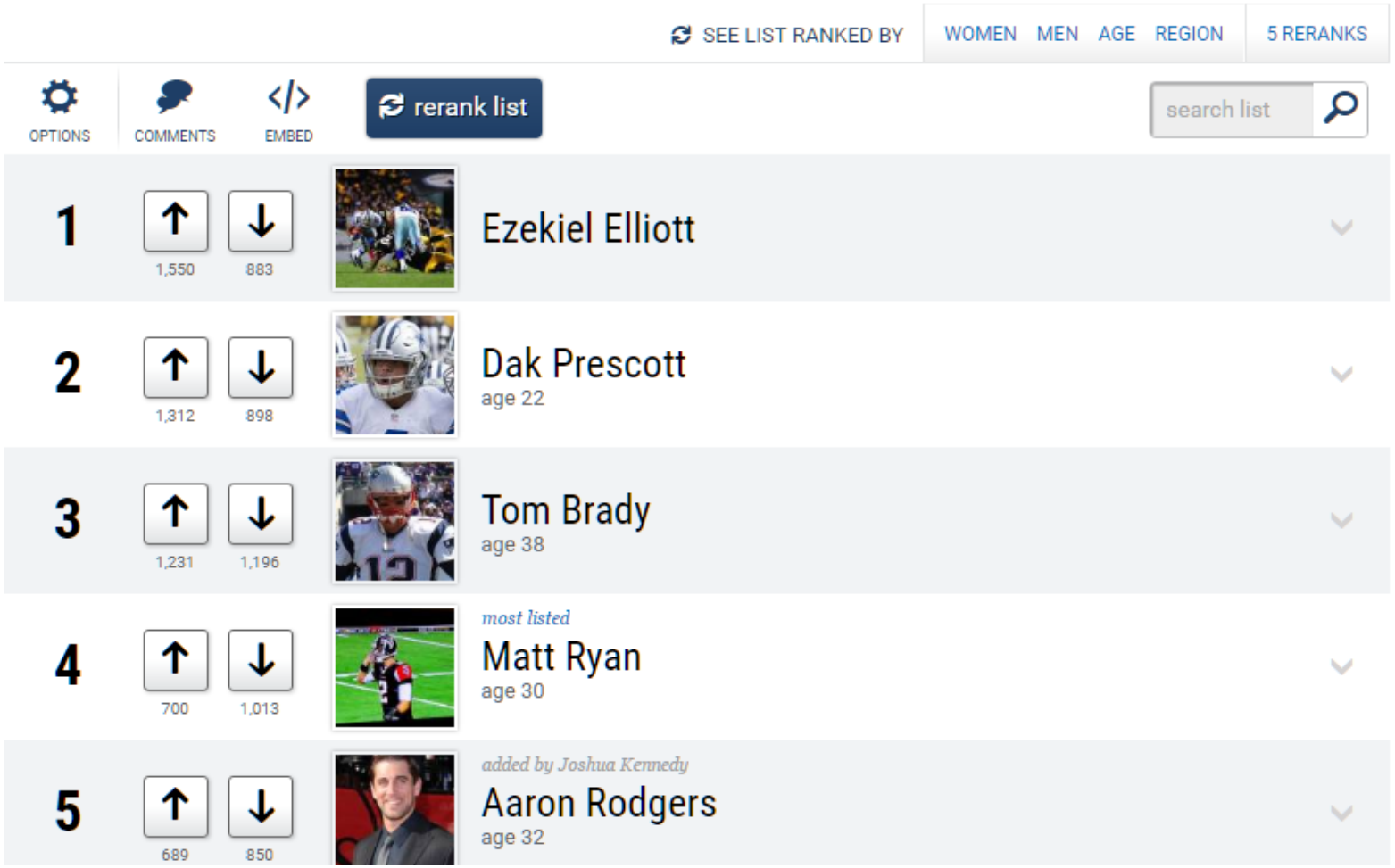
Results

- Find evidence for individual differences, with both incremental and step-change learning
  - can infer learning curves, and Bayes factors

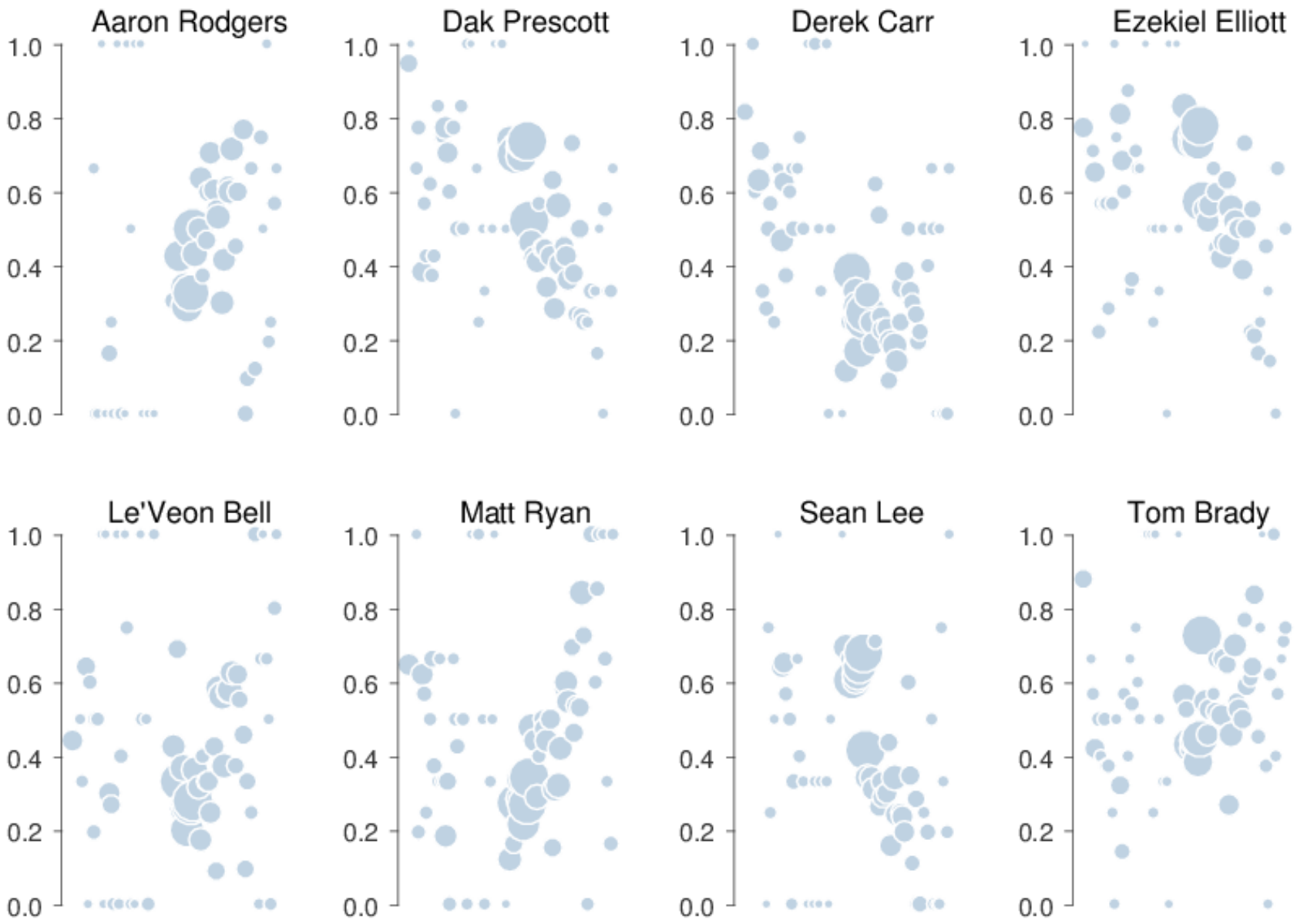


ranker.com NFL 2016-2017 MVP Votes

- List created in November 2016, and received 31,907 votes for 27 different players, up until Matt Ryan won on February 4, 2017

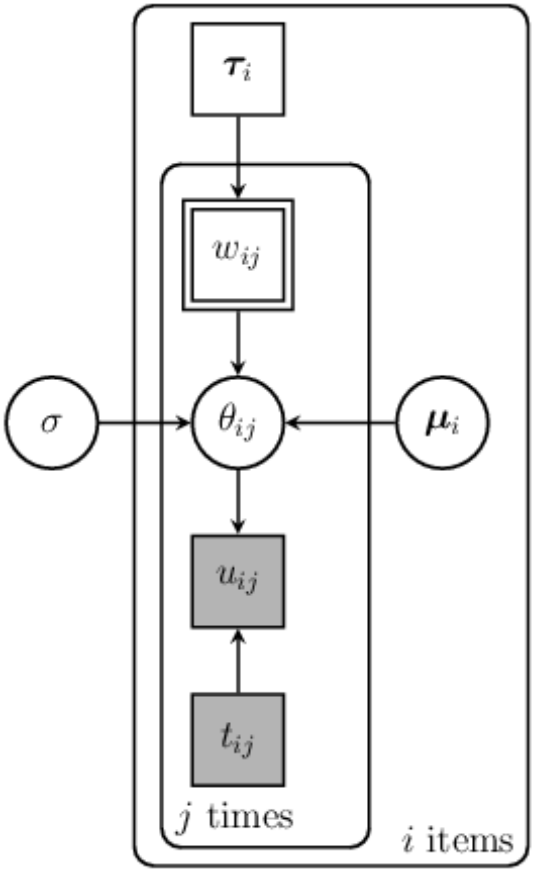


Voting Patterns for Eight Favorites



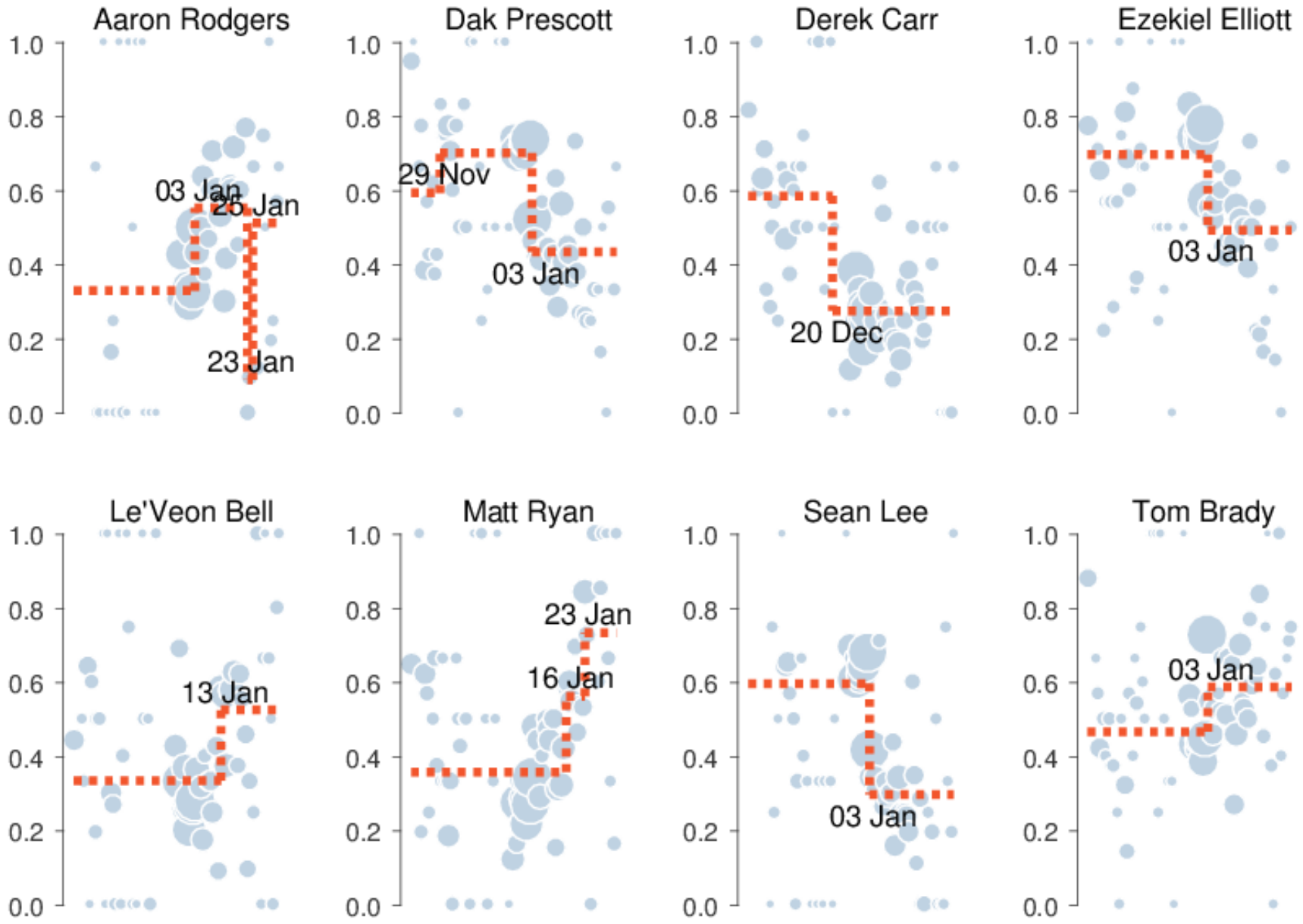
Hierarchical Extension

- The rates within a stage come from an overarching Gaussian
  - assumed the variance is the same for all stages, coming from the different people in the crowd day-to-day



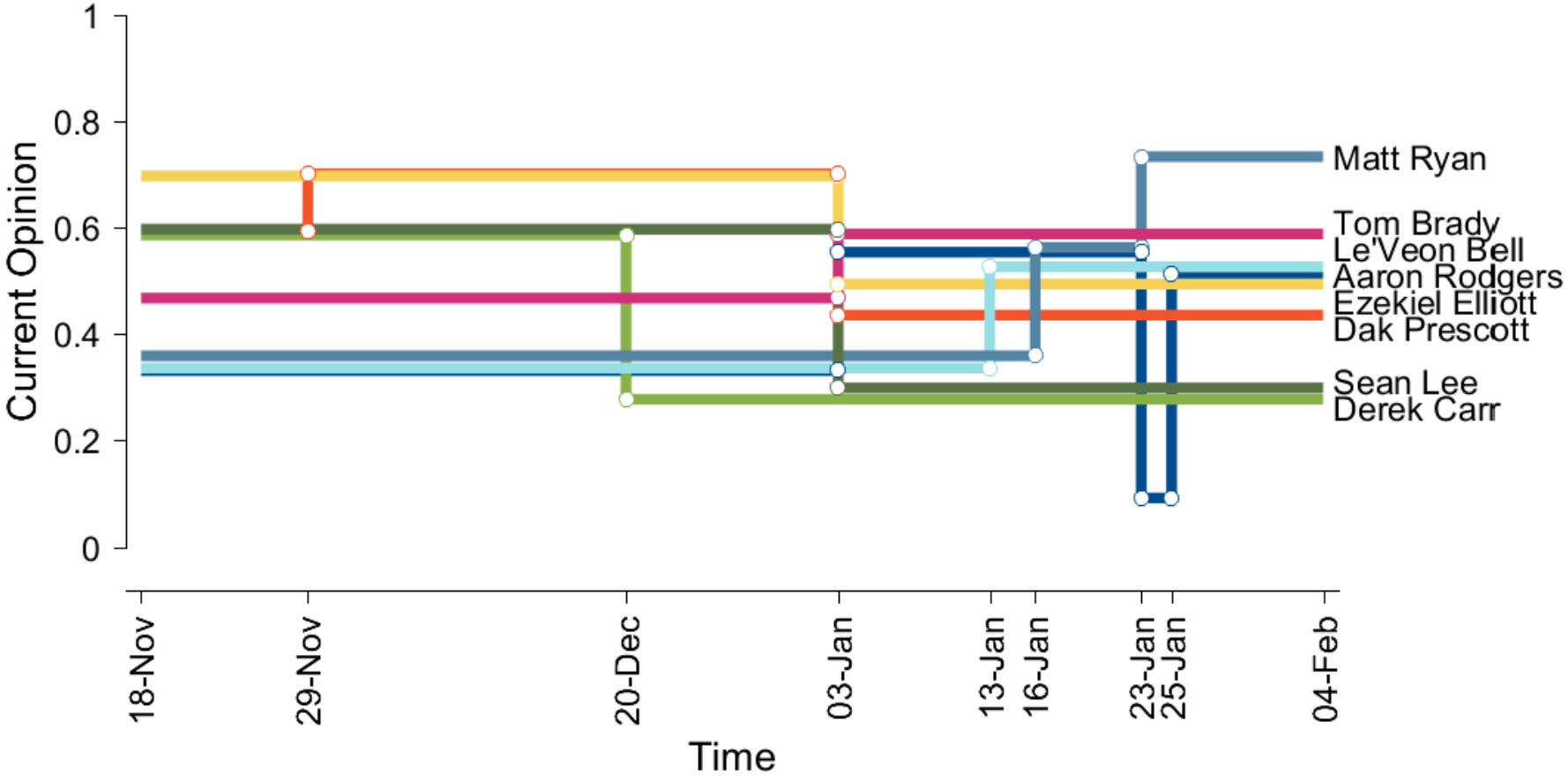
$$
\begin{aligned}
\mu_{ik} &\sim \text{Uniform}(0, 1) \\
\sigma &\sim \text{Gaussian}_{(0,\infty)}\left(\mu', 1/\left(\sigma'\right)^2\right) \\
\tau_{ik} &\sim \text{Categorical}(\overbrace{\frac{n}{2n-1}, \frac{1}{2n-1}, \ldots, \frac{1}{2n-1}}^{n \text{ times}}); \tau_1 \leq \ldots \leq \tau_\gamma \\
w_{ij} &= \textstyle\sum_k \mathcal{I}\left(j \geq \tau_{ik}\right) \\
\theta_{ij} &\sim \text{Gaussian}_{(0,1)}\left(\mu_{i,w_{ij}}, 1/\sigma^2\right) \\
u_{ij} &\sim \text{Binomial}\left(\theta_{ij}, t_{ij}\right)
\end{aligned}
$$

Inferred Step Changes in Opinions



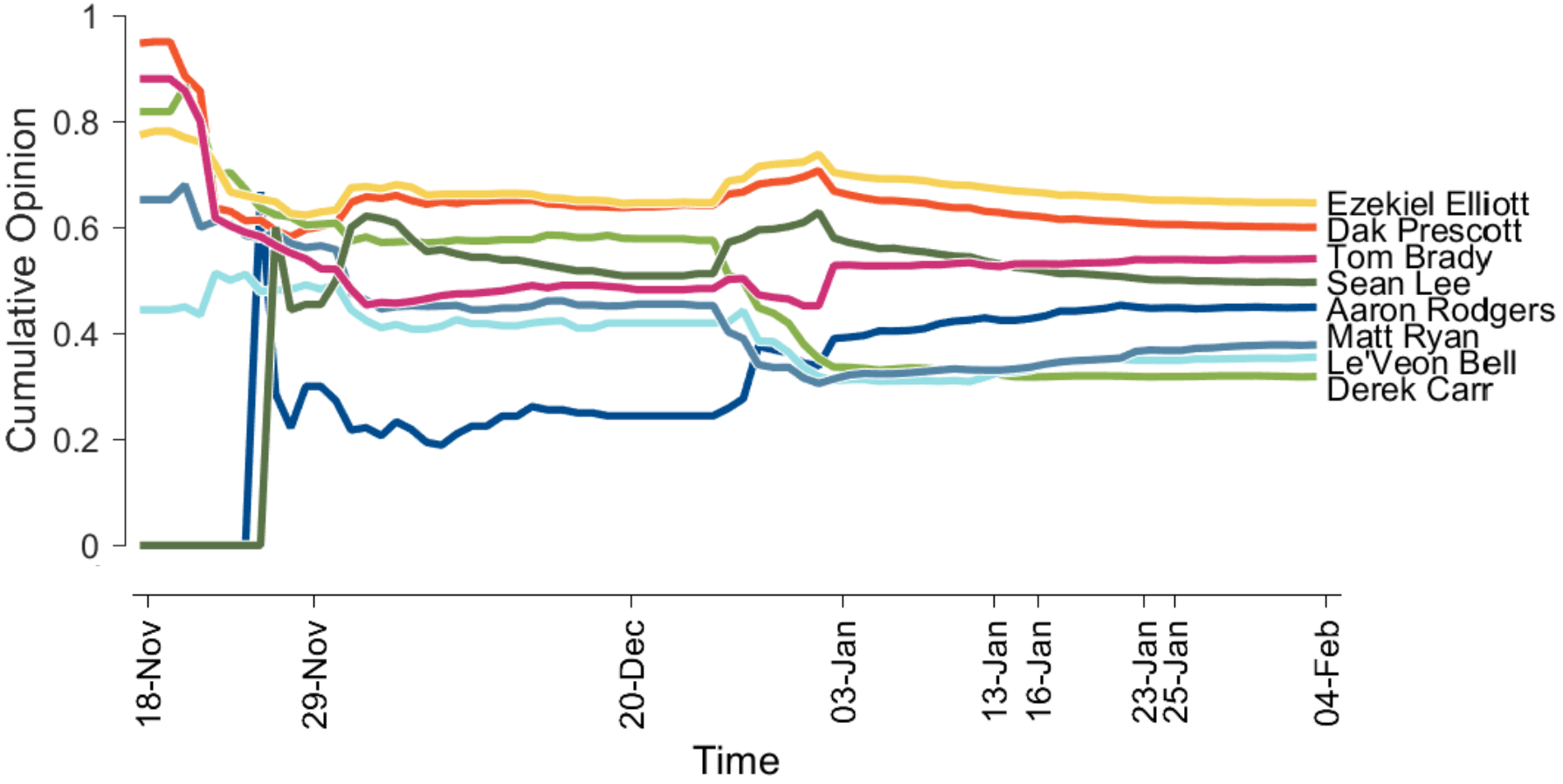
Current Opinion

- At the time of voting, Matt Ryan has the best up-vote proportion, ahead of Tom Brady



Cumulative Opinion

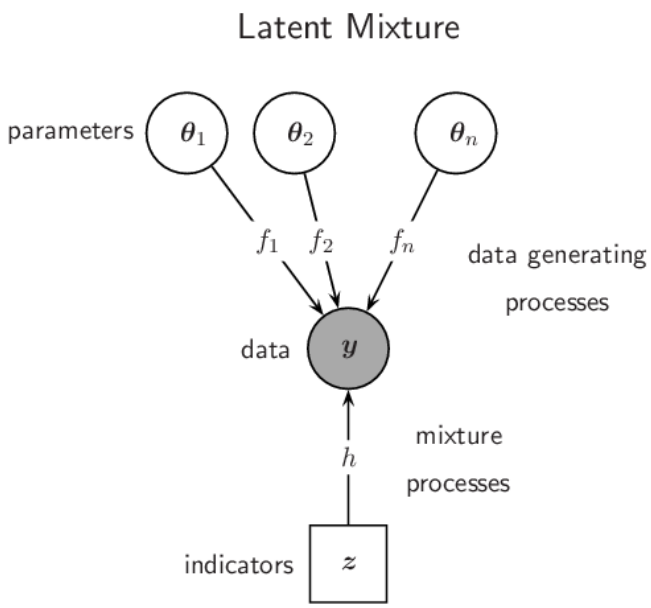
- Ezekiel Elliott had the greatest proportion of up-votes overall

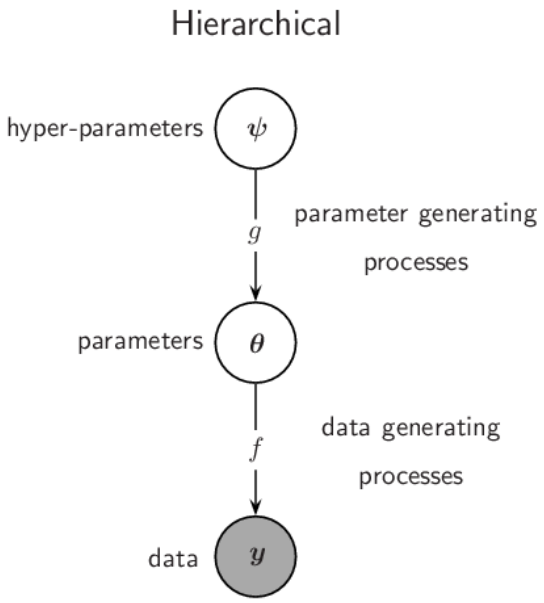


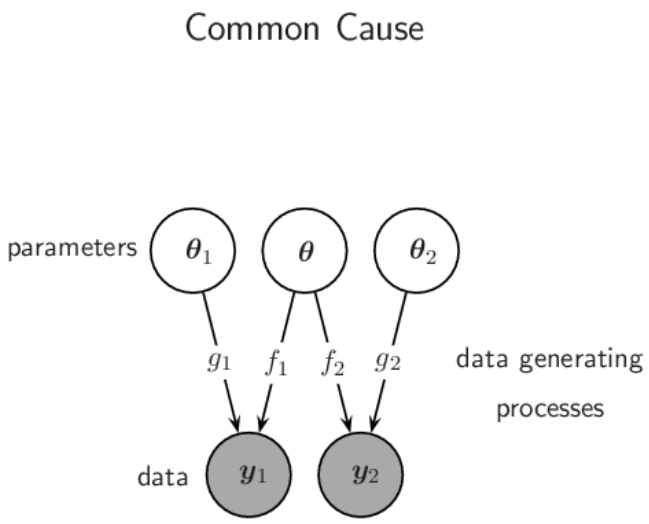
Summary

Bayesian Benefits for Cognitive Modeling

- Bayesian methods allow theorists to develop, evaluate, and use richer generative models of how psychological variables and processes generate behavior
- Bayesian methods afford **theoretical freedom** with **rigorous assessment** and **flexible inferences**

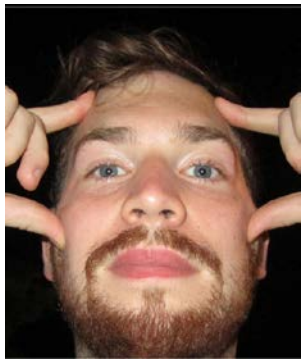






Collaborators

Ravi Iyer
Ranker



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