Multilayer networks continue to gain significant attention in many areas of study, particularly, due to their high utility in modeling interdependent systems such as critical infrastructures, human brain connectome, and socio-environmental ecosystems. However, clustering of multilayer networks, especially, using the information on higher order interactions of the system entities, yet remains in its infancy. We discuss a new topological approach for multilayer network clustering, based on the rationale to group nodes not using the pairwise connectivity patterns or relationships between observations recorded at two individual nodes, but based on how similar in shape their local neighborhoods are at various resolution scales. We quantify shapes of local node neighborhoods using persistence diagrams and then consider either single linkage or k-means forms of topological clustering, which allows us to systematically account for the important heterogeneous higher-order properties of node interactions within and in-between network layers and to integrate information from the node neighbors. In case of topological k-means, we also show that casting it into an empirical risk minimization framework using reproducing kernel Hilbert spaces allows us to derive clustering stability guarantees, similarly to the Euclidean k-means, i.e., property that most existing topological clustering methods lack. We illustrate our topological clustering methods in application to climate-insurance and COVID-19 data.